CURRENT SCHOLARSHIP

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I am finishing a book to be entitled "The Vitushkin Conjecture for Removable Sets ". A compact subset K of the complex plane is said to be *removable for bounded analytic functions* iff every function which is bounded and analytic on a neighborhood of K minus K has an analytic extension across K. Painlevé's problem, posed around 1900, is to give a "geometric" characterization of the removable sets of the complex plane. The problem has proven to be difficult and subtle, resisting progress and solution until recently.

Denjoy's Conjecture from 1909, a special case of Painlevé's problem, states that a compact subset of a rectifiable curve in the complex plane is removable iff it has arc-length measure zero. The last piece of an affirmative solution to this conjecture was put in place in 1977 when Calderón published his celebrated paper dealing with Cauchy integrals on Lipschitz curves. The proof of Denjoy's Conjecture presented in this book is of more recent vintage, utilizing the concept of the curvature of a measure instead of singular integrals.

Indeed, this concept of the curvature of a measure, introduced by Mark Melnikov in a paper from 1995, was crucial to and touched off a long train of progress which culminated in an affirmative solution, the work of many mathematicians, to a conjecture of Vitushkin from 1967 (one can argue about this date). Vitushkin's Conjecture, which expands upon Denjoy's Conjecture and is another special case of Painlevé's problem, states that a compact subset of the complex plane with finite linear Hausdorff measure is removable iff it intersects every rectifiable curve in a set of arc-length measure zero. The main goal of this book is to give a self-contained exposition of this resolution of Vitushkin's Conjecture.

Quite recently Tolsa has proven some very deep results on analytic capacity that extend the Vitushkin Conjecture to those compact subsets of the complex plane with σ-finite linear Hausdorff measure. His work also disposes of Painlevé's problem! It does this by giving an affirmative solution to the following conjecture of Melnikov: A compact subset of the complex plane is removable iff it does not support a measure of linear growth with finite curvature. The book ends with a report on this work.

The chapters of the book are as follows:

1. Removable Sets and Analytic Capacity
2. Removable Sets and Hausdorff Measure
3. Garabedian Duality for Hole-Punch Domains
4. Melnikov and Verdera's Solution to the Denjoy Conjecture
5. Some Measure Theory
6. A Solution to the Vitushkin Conjecture Modulo Two Difficult Results
7. The $T(b)$ Theorem of Nazarov, Treil, and Volberg
8. The Curvature Theorem of Léger
9. A Report on Tolsa's Work Beyond the Vitushkin Conjecture