Sample Problems for Confidence Intervals and Hypothesis Testing

1. Determine whether each of the following statements is true or false.

A) The margin of error for a 95% confidence interval for the mean \( \mu \) increases as the sample size increases.

B) The margin of error for a confidence interval for the mean \( \mu \), based on a specified sample size \( n \), increases as the confidence level decreases.

C) The margin of error for a 95% confidence interval for the mean \( \mu \) decreases as the population standard deviation decreases.

D) The sample size required to obtain a confidence interval of specified margin of error \( m \), increases as the confidence level increases.

Answer: A) False, B) False, C) True, D) True

2. A small New England college has a total of 400 students. The Math SAT is required for admission, and the mean score of all 400 students is 620. The population standard deviation is found to be 60. The formula for a 95% confidence interval yields the interval 640 ± 5.88. Determine whether each of the following statements is true or false.

A) If we repeated this procedure many, many times, only 5% of the 95% confidence intervals would fail to include the mean Math SAT score of the population of all students at this college.

B) The probability that the population mean will fall between 634.12 and 645.88 is 0.95.

C) The interval is incorrect. It is much too narrow.

D) If we repeated this procedure many, many times, \( \bar{x} \) would fall between 634.12 and 645.88 about 95% of the time.

Answer: A) False, B) False, C) False, D) False
3. A sprinkler system is being installed in a large office complex. Based on a series of test runs, a 99\% confidence interval for $\mu$, the average activation time of the sprinkler system (in seconds), is found to be (22, 28). Determine whether each of the following statements is true or false.

A) The 99\% confidence level implies that $P(22 < \mu < 28) = 0.99$.

B) The 99\% confidence level implies that $P(22 < \bar{x} < 28) = 0.99$.

C) The 99\% confidence level implies that 99\% of the sample means ($\bar{x}$) obtained from repeated sampling would fall between 22 and 28.

D) If a 95\% confidence interval were calculated from the same data, (23, 27) would be a possible interval.

Answer: A) False, B) False, C) False, D) True

4. Suppose that the population of the scores of all high school seniors who took the SAT math (SATM) test this year follows a normal distribution with standard deviation $\sigma = 100$. You read a report that says, “On the basis of a simple random sample of 100 high school seniors that took the SATM test this year, a confidence interval for $\mu$ is found to be $512.00 \pm 25.76$.” What was the confidence level used to calculate this confidence interval?

A) 90\%
B) 95\%
C) 99\%
D) 99.9\%

Answer: C

5. To assess the accuracy of a laboratory scale, a standard weight that is known to weigh exactly 1 gram is repeatedly weighed a total of n times and the mean is computed. Suppose the scale readings are normally distributed with unknown mean $\mu$ and standard deviation $\sigma = 0.01$ g. How large should n be so that a 95\% confidence interval for $\mu$ has a margin of error no larger than $\pm 0.0001$?

A) n = 100.
B) n = 196.
C) n = 10000
D) n = 38416.

Answer: D
Use the following to answer questions 6 and 7:

The heights of a simple random sample of 400 male high school sophomores in a Midwestern state are measured. The sample mean is \( \bar{x} = 66.2 \) inches. Suppose that the heights of male high school sophomores follow a normal distribution with standard deviation \( \sigma = 4.1 \) inches.

6. What is a 95% confidence interval for \( \mu \)?

A) (58.16, 74.24)
B) (59.46, 72.94)
C) (65.80, 66.60)
D) (65.86, 66.54)

Answer:  
C

7. Suppose the heights of a simple random sample of 100 male sophomores were measured rather than 400. Which of the following statements is true?

A) The margin of error for the 95% confidence interval would increase.
B) The margin of error for the 95% confidence interval would decrease.
C) The margin of error for the 95% confidence interval would stay the same, because the level of confidence has not changed.
D) The standard deviation \( \sigma \) would decrease.

Answer:  
A

8. The tail area above a test statistic value of \( z = 1.812 \) is 0.035. Determine whether each of the following statements is true or false.

A) If the alternative hypothesis is of the form \( H_a: \mu > \mu_0 \), the data are statistically significant at significance level \( \alpha = 0.05 \).
B) If the alternative hypothesis is of the form \( H_a: \mu > \mu_0 \), the data are statistically significant at significance level \( \alpha = 0.10 \).
C) If the alternative hypothesis is of the form \( H_a: \mu \neq \mu_0 \), the data are statistically significant at significance level \( \alpha = 0.05 \).
D) If the alternative hypothesis is of the form \( H_a: \mu \neq \mu_0 \), the data are statistically significant at significance level \( \alpha = 0.10 \).

Answer:  
A) True, B) True, C) False, D) True

Use the following to answer questions 9 and 10:
The Survey of Study Habits and Attitudes (SSHA) is a psychological test that measures the motivation, attitude, and study habits of college students. Scores range from 0 to 200 and follow (approximately) a normal distribution with mean 115 and standard deviation 25. You suspect that incoming freshman at your school have a mean $\mu$, which is different from 115 because they are often excited yet anxious about entering college. To test your suspicion, you decide to test the hypotheses $H_0: \mu = 115$ versus $H_a: \mu \neq 115$. You give the SSHA to 25 incoming freshman and find their mean score to be 116.2.

9. What is the value of the test statistic?
   A) $z = 0.048$  C) $z = 1.2$
   B) $z = 0.24$  D) $z = 1.96$
   Answer: B

10. What is the value of the $P$-value?
    A) 0.1151  C) 0.4052
    B) 0.2302  D) 0.8104
    Answer: D

Use the following to answer questions 11 and 12:

The nicotine content in cigarettes of a certain brand is normally distributed with standard deviation $\sigma = 0.1$ milligrams. The brand advertises that the mean nicotine content of their cigarettes is $\mu = 1.5$, but measurements on a random sample of 100 cigarettes of this brand gave a mean of $\bar{x} = 1.53$. Is this evidence that the mean nicotine content is actually higher than advertised? To answer this, test the hypotheses $H_0: \mu = 1.5$ versus $H_a: \mu > 1.5$.

10. What is the value of the $P$-value?
    A) 0  C) 0.3821
    B) 0.0013  D) 0.9987
    Answer: B

Determine whether each of the following statements is true or false.

11. A) The probability that $H_0$ is true is 0.05.
    B) The data were statistically significant at $\alpha = 0.05$.
    C) At the 5% significance level, $H_a$ should be rejected.
    D) Even if nicotine content in cigarettes of this brand were not quite normally distributed, the test would still be valid.
    Answer: A) False, B) True, C) False, D) True

Use the following to answer questions 12 and 13:

The time needed for college students to complete a certain paper-and-pencil maze follows a normal distribution with a mean of 30 seconds and a standard deviation of 3 seconds. You wish to see if the mean time $\mu$ is changed by vigorous exercise, so you have a group of nine college students exercise vigorously for 30 minutes and then complete the maze. Assume that $\sigma$ remains unchanged at 3 seconds. The hypotheses you decide to test are $H_0: \mu = 30$ versus $H_a: \mu \neq 30$. 

12. Suppose it takes the nine students an average of $\bar{x} = 32.05$ seconds to complete the maze. At the 1% significance level, what can you conclude?
A) $H_0$ should be rejected because the $P$-value is less than 0.01.
B) $H_0$ should not be rejected because the $P$-value is greater than 0.01.
C) $H_a$ should be rejected because the $P$-value is less than 0.01.
D) $H_a$ should not be rejected because the $P$-value is greater than 0.01.
Answer: B

13. Suppose you compute the average time $\bar{x}$ that it takes these students to complete the maze and you find that the results are significant at the 5% level. What can you conclude?
A) The test would also be significant at the 10% level.
B) The test would also be significant at the 1% level.
C) Both of the above.
D) None of the above.
Answer: A

Use the following to answer questions 14-16:

A major car manufacturer wants to test a new engine to determine if it meets new air pollution standards. The mean emission $\mu$ of all engines of this type must be approximately 20 parts per million of carbon. If it is higher than that, they will have to redesign parts of the engine. Ten engines are manufactured for testing purposes and the emission level of each is determined. Based on data collected over the years from a variety of engines, it seems reasonable to assume that emission levels are roughly normally distributed with $\sigma = 3$.

14. What are the appropriate null and alternative hypotheses?
A) $H_0$: $\mu = 20$ vs. $H_a$: $\mu < 20$
B) $H_0$: $\mu = 20$ vs. $H_a$: $\mu \neq 20$
C) $H_0$: $\mu = 20$ vs. $H_a$: $\mu > 20$
Answer: C

15. The data result in an average of 22 parts per million. What is the value of the test statistic?
A) $z = -2.11$
B) $z = 0.667$
C) $z = 2.11$
D) $z = 6.67$
Answer: C
16. What is the value of the $P$-value?
   A) Less than 0.0001.
   B) 0.0175
   C) 0.2525
   D) 0.9825

Answer: B