# STT 861 (Fall 2019): Homework 1

This homework will be collected at the start of the lecture of **Wednesday Sep 11, 2019**.

**Question 1.** Let $A$, $B$ and $C$ be three events and $P(\cdot)$ be a probability. Show that

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C).$$

**Question 2.** A sequence of events $\{A_n : n = 1, 2, \ldots\}$ is said to increase to $A$ if

$$\text{(i) } A_1 \subseteq A_2 \subseteq A_3 \subseteq \cdots, \qquad \text{and} \qquad \text{(ii) } A = \cup_{n=1}^{\infty} A_n.$$

We denote $A_n \uparrow A$.
Define $B_1 = A_1$, and $B_n = A_n \setminus A_{n-1}$, for $n = 2, 3, \ldots$. Show the following:

(a) Show that $B_1, B_2, \ldots$ are pairwise disjoint, i.e. for all $i \neq j$, $B_i \cap B_j = \emptyset$.

(b) $A_n = \cup_{j=1}^{n} B_j$, for any $n = 1, 2, 3, \ldots$.

(c) $A = \cup_{j=1}^{\infty} B_j$.

(d) $P(A) = \lim_{n\to\infty} P(A_n)$.

**Question 3.** Suppose $P(A^c) = 0.2$ and $P(B) = 0.3$. Can $A$ and $B$ be disjoint? Justify.