In this lecture, we discuss some important continuous distributions and also a graphical method to check if the data is from a continuous distribution.

**Gamma Function**
The gamma function $\Gamma(\alpha)$, for $\alpha > 0$, is defined by

$$\Gamma(\alpha) = \int_{0}^{\infty} x^{\alpha-1} e^{-x} dx.$$ 

**Gamma Distribution $G(\alpha, \beta)$**
A continuous random variable $X$ is said to have a gamma distribution, denoted by $G(\alpha, \beta)$, if the pdf of $X$ is

$$f(x|\alpha, \beta) = \begin{cases} \frac{1}{\beta^\alpha \Gamma(\alpha)} x^{\alpha-1} e^{-x/\beta}, & x \geq 0 \\ 0, & \text{otherwise}. \end{cases}$$

where $\alpha > 0$ and $\beta > 0$. 
Here, $E(X) = \alpha \beta$ and $V(X) = \alpha \beta^2$. The gamma distribution $G(\alpha, 1)$ is called **standard gamma distribution**.

The cdf $F(x)$ does not admit closed form. However, for the standard gamma distribution, $F(x)$ can be found using the Table A.4 on p. A-8.

Note also that if $X \sim G(\alpha, \beta)$, then $\frac{X}{\beta} \sim G(\alpha, 1)$. Using this result, the probabilities of the gamma distribution can be calculated, using the Table A.4.

The density functions of some gamma distributions are given below.
Continuous Distributions and Probability Plots

Figure: Gamma densities

- $\alpha = 5$, $\beta = 1$
- $\alpha = 4$, $\beta = 0.9$
- $\alpha = 3$, $\beta = 0.8$
- $\alpha = 6$, $\beta = 1.3$
Example 1 (Eg. 4.24)
Suppose the survival time $X$ in weeks of a randomly selected male mouse exposed of 240 rads of gamma radiation has a gamma distribution with $\alpha = 8$ and $\beta = 15$. What is the probability that a mouse survives between 60 and 120 weeks?

Note here that $X \sim G(8, 15)$. Let $Y = X/15$ so that $Y \sim G(8, 1)$. 

Hence,

$$P(60 < X < 120) = P(4 < Y < 8) = 0.547 - 0.051 = 0.496,$$

using Table A-4.
Example 2 (Ex 66): Suppose the time spent by a randomly selected student who uses a terminal connected to a local time-sharing computer facility has a gamma distribution with mean 20 min and variance 80 min$^2$.

(a) What are the values of $\alpha$ and $\beta$?
(b) What is the probability that a student uses the terminal for at most 24 min?
(c) What is the probability that a student spends between 20 and 40 min using the terminal?

Solution: Let $F(x|\alpha, \beta)$ denote the cdf of $X \sim G(\alpha, \beta)$.

(a) $\mu = 20$, $\sigma^2 = 80 \Rightarrow \alpha \beta = 20$, $\alpha \beta^2 = 80 \Rightarrow \beta = \frac{80}{20} = 4$, $\alpha = 5$.
(b) $P(X \leq 24) = F\left(\frac{24}{4}|5, 1\right) = F(6|5, 1) = 0.715$ (Using table A.4)
(c) $P(20 \leq X \leq 40) = F(10|5, 1) - F(5|5, 1) = 0.411$. 
Continuous Distributions and Probability Plots

Exponential Distribution
The gamma distribution with $\alpha = 1$ and $\beta = 1/\lambda$ is also called an exponential distribution with parameter $\lambda > 0$. It has the pdf

$$f(x|\lambda) = \begin{cases} \lambda e^{-\lambda x}, & x \geq 0 \\ 0, & \text{otherwise.} \end{cases}$$

If $X$ is an exponential random variable with parameter $\lambda$, then

$$E(X) = \frac{1}{\lambda} \quad \text{and} \quad V(X) = \frac{1}{\lambda^2}.$$  

The cdf is $F(x) = \int_0^x \lambda e^{-\lambda t} dt = 1 - e^{-\lambda x}, \quad x > 0$. Also, the reliability function of $X$ is

$$R(t) = P(X > t) = e^{-\lambda t}, \quad t > 0.$$  

The Chi-squared Distribution
The distribution $G(\gamma/2, 2)$ is called $\chi^2$ distribution with $\gamma$ degrees of freedom. That is, $\chi^2_{\nu} \equiv G(\nu/2, 2)$.
Continuous Distributions and Probability Plots

Figure: Exponential densities

- Black line: $\lambda = 0.5$
- Red line: $\lambda = 1$
- Blue line: $\lambda = 2$

(P. Vellaisamy: MSU-STT-351-Sum-19A) Probability & Statistics for Engineers 8/43
Example 3 (Ex. 71)

(a) The event \( \{ X^2 \leq y \} \) is equivalent to what event involving \( X \) itself?

(b) If \( X \sim N(0, 1) \), show that, using Part (a), \( X^2 \sim \chi^2_1 \).

Solution. Let \( f_X(t) \) be the pdf and \( F_X(t) \) be the cdf of \( X \). Let \( Y = X^2 \). Then

(a) \( \{ X^2 \leq y \} = \{ Y \leq y \} = \{ -\sqrt{y} \leq X \leq \sqrt{y} \} \), since \( y > 0 \).

(b) Note \( F_Y(y) = P(X^2 \leq y) = F_X(\sqrt{y}) - F_X(-\sqrt{y}) \). Differentiating wrt \( y \),

\[
f_Y(y) = f_X(\sqrt{y})\left(\frac{1}{2\sqrt{y}}\right) - f_X(-\sqrt{y})\left(-\frac{1}{2\sqrt{y}}\right)
= \frac{1}{\sqrt{2\pi}}e^{-y/2}\left(\frac{1}{2}y^{-1/2}\right) + \frac{1}{\sqrt{2\pi}}e^{-y/2}\left(\frac{1}{2}y^{-1/2}\right)
= \frac{1}{\sqrt{2\pi}}e^{-y/2}y^{-1/2} = \frac{1}{2^{1/2}\Gamma(1/2)}e^{-y/2}y^{-1/2},
\]

which is \( G(1/2, 2) = \chi^2_1 \). Note we have used the fact \( f_X(x) = \frac{1}{\sqrt{2\pi}}e^{-x^2/2} \).
Example 4 (Ex 69 b, c): A system consists of five identical components connected in series as shown:

The entire system fails if one component fails. Suppose each component has a lifetime that is exponentially distributed with $\lambda = 0.01$ and that components fail independently.

Let $X_i$ denote the life-time of the $i$-th component. Then $X_1, \ldots, X_5$ are independent rvs. Also, $(X_i > t)$ denotes the event survives till time $t$. Note $X = \min\{X_1, X_2, \ldots, X_n\}$ denotes the life-time of the system.
(a) Compute \( P(X \geq t) \), \( F(t) = P(X \leq t) \) and the pdf \( f(t) \) of \( X \). What type of distribution does \( X \) have?

(b) Suppose there are \( n \) components, each having exponential lifetime with parameter \( \lambda \). What type of distribution does \( X \) have?

**Solution:** Note first that \( X = \min\{X_1, X_2, \ldots, X_n\} \) denotes the life-time of the system. Now

(a) \[ P(X \geq t) = P(X_1 \geq t)P(X_2 \geq t) \ldots P(X_5 \geq t) = (e^{-\lambda t})^5 = e^{-0.05t}. \]
So,

\[ F_X(t) = P(X \leq t) = 1 - e^{-0.05t}; \quad f_X(t) = 0.05e^{-0.05t} \text{ for } t \geq 0. \]

Thus, \( X \) also has an exponential distribution with parameter \( \lambda = 0.05 \).

(b) By the same reasoning, \( P(X \leq t) = 1 - e^{-n\lambda t} \), so \( X \) has an exponential distribution with parameter \( n\lambda \).
Continuous Distributions and Probability Plots

Weibull Distribution: $W(\alpha, \beta)$

This is an extension of exponential distribution.

**Definition:** A continuous variable $X$ follows $W(\alpha, \beta)$ if its density is

$$f(x; \alpha, \beta) = \begin{cases} \frac{\alpha}{\beta^\alpha} e^{-(\frac{x}{\beta})^\alpha} x^{\alpha - 1}, & x > 0 \\ 0, & \text{otherwise} \end{cases}$$

(i) Different values of $\alpha$ and $\beta$ give positively and negatively skewed distributions.

(ii)

$$\mu = E(X) = \frac{\beta}{\alpha} \Gamma(1/\alpha); \sigma^2 = V(X) = \frac{2\beta^2}{\alpha} \Gamma\left(\frac{2}{\alpha}\right).$$
Continuous Distributions and Probability Plots

Figure: Weibull densities

- Black line: $\alpha = 2$, $\beta = 2$
- Red line: $\alpha = 2$, $\beta = 4$
- Blue line: $\alpha = 1.5$, $\beta = 5$
- Cyan line: $\alpha = 2.5$, $\beta = 6$
(iii) \[
P(X \leq t) = \int_{0}^{t} f(x|\alpha, \beta) \, dx \\
= \frac{\alpha}{\beta^\alpha} \int_{0}^{t} e^{-\left(\frac{x}{\beta}\right)^\alpha} x^{\alpha-1} \, dx \\
= \int_{0}^{\left(\frac{t}{\beta}\right)^\alpha} e^{-y} \, dy,
\]
using the transformation \( y = \left(\frac{x}{\beta}\right)^\alpha \) so that \( dy = \frac{\alpha}{\beta^\alpha} x^{\alpha-1} \, dx \). Thus, we get

\[
P(X \leq t) = \left[ -e^{-y} \right]_{0}^{\left(\frac{t}{\beta}\right)^\alpha} = 1 - e^{-\left(\frac{t}{\beta}\right)^\alpha}.
\]

Therefore,

\[
P(X > t) = e^{-\left(\frac{t}{\beta}\right)^\alpha}, \quad t > 0.
\]

(iv) If \( X \sim \text{W}(\alpha, \beta) \), then \( Y = kX \sim \text{W}(\alpha, k\beta) \).
Example 5. Suppose $X \sim W(2, 10)$. Then

(i) $P(X < 10) = 1 - e^{\left(-\frac{10}{10}\right)^2} = 1 - e^{-1} \approx 0.632$.

(ii) Suppose $c$ is $P(X < c) = 0.95$. Then

$$1 - e^{\left(-\frac{c}{10}\right)^2} = 0.95$$

Solving for $c$, we get $c \approx 17.3$ which is the 95-th percentile.
Example 6(Ex 76):
Let $X$ follow Weibull distribution with parameters $\alpha = 2$ and $\beta = 3$.
(a) What is median lifetime of such tubes?
(b) If $X$ has a Weibull distribution with the cdf form expression, obtain a general expression for the $(100p)$th percentile of the distribution.

**Solution** The median $m$ satisfies
(a) $F(m) = 1 - e^{-(m/3)^2} \Rightarrow e^{-m^2/9} = 0.5 \Rightarrow m^2 = -9 \ln(.5) = 6.2383 \Rightarrow m = 2.50$

(b)
$p = F(x_p) = 1 - e^{(x_p/\beta)\alpha} \Rightarrow (x_p/\beta)^\alpha = -\ln(1-p) \Rightarrow x_p = \beta[-\ln(1-p)]^{1/\alpha}.$
Nest, we discuss a variation of the normal distribution.

**The Lognormal Distribution**

**Definition:** A variable $X > 0$ is said to follow $LN(\mu, \sigma)$ if $\ln(X) \sim N(\mu, \sigma)$. Its density is

$$f(x; \mu, \sigma) = \begin{cases} \frac{1}{\sqrt{2\pi} \sigma} \frac{1}{x} e^{\frac{1}{2\sigma^2} (\ln x - \mu)^2}, & x > 0 \\ 0, & \text{otherwise} \end{cases}$$

Note that $\ln(X)$ (and not $X$) follows normal distribution with mean $\mu$ and variance $\sigma^2$.

The graphs of some lognormal densities are given below.
Figure: Lognormal densities

- $\mu = 1.0, \sigma = 1.0$
- $\mu = 1.5, \sigma = 0.5$
- $\mu = 2.0, \sigma = 1.0$
- $\mu = 3.0, \sigma = 1.5$
Facts:
(i) It is easy to check
\[
\int_{-\infty}^{\infty} f(y|\mu, \sigma) \, dy = 1,
\]
using the substitution \( y = \ln(x) \).

(ii) If \( X \sim LN(\mu, \sigma) \), then \( Y = \ln(X) \sim N(\mu, \sigma) \). Hence,
\[
P(X \leq x) = P[\ln(X) \leq \ln(x)] = P\left(Z \leq \frac{\ln(x) - \mu}{\sigma}\right) = \Phi\left(\frac{\ln(x) - \mu}{\sigma}\right).
\]

(iii) \( E(X) = e^{\mu + \frac{\sigma^2}{2}} \); \( V(X) = e^{2\mu + \sigma^2}(e^{\sigma^2} - 1) \).

The above facts are used to compute probabilities.
Example 7 (Ex 79): Let the rv $X$ denote the hourly median power (in decibels) of received radio signals transmitted between two cities. The authors of an article argue that the lognormal distribution provides a reasonable probability model for $X$.

If the parameter values are $\mu = 3.5$ and $\sigma = 1.2$, calculate the following:

(a) The mean value and standard deviation of received power.

(b) The probability that received power is between 50 and 250dB.

(c) The probability that $X$ is less than its mean value. Why is this probability not 0.5?
Solution: Given that \( X \sim LN(3.5, 1.2) \). Then

(a):

\[
E(X) = e^{3.5 + (1.2)^2/2} = 68.0335; \\
V(X) = e^{2(3.5) + (1.2)^2} \left( e^{(1.2)^2 - 1} \right) = 14907.168 \\
\sigma_X = 122.0949.
\]

(b) : \( P(50 \leq X \leq 250) = P\left(Z \leq \frac{\ln(250) - 3.5}{1.2}\right) - P\left(Z \leq \frac{\ln(50) - 3.5}{1.2}\right) = P(Z \leq 1.68) - P(Z \leq .34) = 0.9535 - 0.6331 = 0.3204. \) (use table)

(c) \( P(X \leq 68.0335) = P\left(Z \leq \frac{\ln(68.0335) - 3.5}{1.2}\right) = P(Z \leq .60) = 0.7257. \) The lognormal distribution is not a symmetric distribution.
Example 8. The r.v. $X = \text{modulus of elasticity (MOE)}$ of a certain system is observed, in a study, to follow $\text{LN}(0.375, 0.25)$. Find

(i) $P(X < 3)$

(ii) $P(1 < X < 2)$

(iii) For what value of $c$, only 1% of systems have $X > c$? Note, $c$ is nothing but the 99th percentile.

Solution:

(i) Note $X < 3 \iff Y = \ln(X) < \ln(3) = 1.0986$, where $Y \sim \text{N}(0.375, 0.25)$

Therefore,

$$Y < 1.098 \iff Z = \frac{Y - 0.375}{0.25} < \frac{1.098 - 0.375}{0.25} = 2.89,$$

where $Z \sim \text{N}(0, 1)$.

From normal tables, $P(Z < 2.89) = 0.998$. Hence, $P(X < 3) = 0.998$ (or 99.8%).
(ii) Similarly,

\[ 1 < X < 2 \iff \ln(1) < \ln(X) < \ln(2) \]

\[ \iff \ln(1) < Y < \ln(2) \]

\[ \iff \frac{\ln(1) - 0.375}{0.25} < \frac{Y - 0.375}{0.25} < \frac{\ln(2) - 0.375}{0.25} \]

Hence, \( P(1 < X < 2) = P(-1.512 < Z < 1.272) \).

Now

\[ P(Z < 1.272) = 0.8984; \quad P(Z < -1.512) = 0.0653. \text{ (see normal table)} \]

\[ P(-1.512 < Z < 1.272) = 0.8331. \]

Hence,

\[ P(1 < X < 2) = 83.3\% \]
(iii) Need to find \( c \) such that

\[
P(X > c) = 0.01 \Leftrightarrow P(X \leq c) = 0.99.
\]

That is, need to find \( c \) such that

\[
P\left[ \frac{\ln(X) - 0.375}{0.25} \leq \frac{\ln(c) - 0.375}{0.25} \right] = 0.99
\]

That is, \( c \) is such that

\[
P \left( Z \leq \frac{\ln(c) - 0.375}{0.25} \right) = 0.99
\]

From normal table, 99th percentile of \( Z \) is 2.33. Therefore,

\[
\frac{\ln(c) - 0.375}{0.25} = 2.33
\]

Solving \( \ln(c) = 0.9575 \), we get \( c = 2.605 \).
Quantile (or Probability) Plots

The probability plots can be used to check if the data $x_1, \ldots, x_n$ is from a particular distribution.

The basic idea is to compare sample quantiles (percentiles) with population quantiles (percentiles).

**Definition:**

(i) Order the observations as $x(1) \leq x(2) \leq \ldots \leq x(n)$. Then,

(ii) $x(i)$ may be called $\left(\frac{i}{n}\right)$-th or technically $\left(\frac{i-0.5}{n}\right)$-th sample quantile.
Example 9

Let \( n = 10 \) and \( x_1, x_2, \ldots, x_{10} \) be a random sample and \( x(1) \leq x(2) \leq \ldots \leq x(n) \) be their ordered values. Then, sample quantiles are

\[
x(1) \text{ is } \frac{1 - 0.5}{10} = 0.5 \frac{10}{10} = 0.5\text{th quantile (or 5th percentile)}
\]

\[
x(2) \text{ is } \frac{2 - 0.5}{10} = 0.15 \frac{10}{10} = 0.15\text{th quantile (or 15th percentile)}
\]

\[
\vdots
\]

\[
x(10) \text{ is } \frac{10 - 0.5}{10} = 0.95 \frac{10}{10} = 0.95\text{th quantile (or 95th percentile)}
\]

The basic ideas of a quantile plot are summarized below.
Continuous Distributions and Probability Plots

Procedure of Probability (Quantile) Plot

Data: $x_1, x_2, \ldots, x_n$.

Order: $x_{(1)} \leq x_{(2)} \leq \ldots \leq x_{(n)}$.

Compute: $\left( \frac{0.5}{n} \right), \left( \frac{1.5}{n} \right), \ldots, \left( \frac{n-0.5}{n} \right)$

Population quantiles: $\eta_{\frac{0.5}{n}}, \eta_{\frac{1.5}{n}}, \ldots, \eta_{\frac{n-0.5}{n}}$,

If the observations are basically from the population with density $f(x)$, then $x_{(i)} \approx \eta_{\left( \frac{i-0.5}{n} \right)}$. This is the basic idea.

Plot the points: $\left( \eta_{\frac{0.5}{n}}, x_{(1)} \right), \left( \eta_{\frac{1.5}{n}}, x_{(2)} \right), \ldots, \left( \eta_{\frac{n-0.5}{n}}, x_{(n)} \right)$

If the graph is approximately linear, then the data $x_1, \ldots, x_n$ are from density $f(x)$. 
1. Normal Probability Plot

Normal probability plot is used to check if the data $y_1, \ldots, y_n$ are from $N(\mu, \sigma)$:

(i) Order: $y_{(1)} \leq \ldots \leq y_{(n)}$.

(ii) Standardize: $z_{(1)} = \frac{y_{(1)} - \mu}{\sigma}, \ldots, z_{(n)} = \frac{y_{(n)} - \mu}{\sigma}$

(iii) Let $p_1 = \frac{1-0.5}{n}, p_2 = \frac{2-0.5}{n}, \ldots, p_n = \frac{n-0.5}{n}$.

(iv) Find population percentiles of $N(0, 1)$: $\eta_{p_1}, \eta_{p_2}, \ldots, \eta_{p_n}$.

(v) Plot the points: $(\eta_{p_1}, z_{(1)}), (\eta_{p_2}, z_{(2)}), \ldots, (\eta_{p_n}, z_{(n)})$.

If the graph is approximately linear, then sample is from the normal distribution.
Example 10.

Check if the data 9.9, 4.8, 8.1, 15.2, 12.1 are from \(N(10, 4)\).

Ordered data \(x_{(i)}: 4.8, 8.1, 9.9, 12.1, 15.2\)

Normalized data: \(z_{(i)} = \frac{x_{(i)} - 10}{4} : -1.3, -0.475, -0.025, 0.525, 1.3\).

Also, here \(n = 5\) and \(p_1 = \frac{5}{5} = .1; p_2 = \frac{1.5}{5} = .3; p_3 = \frac{2.5}{5} = .5; p_4 = \frac{3.5}{5} = .7; p_5 = \frac{4.5}{5} = .9;\)

From normal table (without interpolation)
\(\eta_{1} = -1.28; \eta_{3} = -0.52; \eta_{5} = 0; \eta_{7} = 0.53; \eta_{9} = 1.29\)

Draw or graph the points \((\eta_{p_i}, z_{(i)})\):
\((-1.28, -1.3), (-0.52, -0.475), (0, -0.025), (0.53, 0.525), (1.29, 1.3).\)
Continuous Distributions and Probability Plots

The plot is given below:

Figure: Scatter plot of $z(i)$ vs $\eta_{p_i}$.

Since the graph is approximately linear, we conclude that the sample is from $N(10, 4)$. 
Example 11. The following data refers to the coating thickness of a paint: 0.83, 0.88, 0.88, 1.04, 1.09, 1.12, 1.29, 1.31, 1.48, 1.49, 1.59, 1.62, 1.65, 1.71, 1.76, 1.83

Can you assume that the above data comes from normal distribution?

Note also that the mean and the standard deviation may be taken to be the corresponding sample quantities, as they are not mentioned in the problem.

The quantile plot for the above-data obtained from Minitab Package is given below:

Since the graph is roughly linear, we conclude that the coating thickness of a paint follows normal.
Example 12: Consider the following normal probability plot.

It is clear that the sample is not from the normal distribution.
2. Lognormal Quantile Plot

If \( X \sim LN(\mu, \sigma) \), then \( \ln(X_1), \ldots, \ln(X_n) \) are from \( N(\mu, \sigma) \). Therefore, check if the data \( \ln(X_1), \ldots, \ln(X_n) \) are from \( N(\mu, \sigma) \), using normal quantile plot.

**Example 12.** Consider the following sample of hourly median power readings of radio signals:

\[
2.7, 5.4, 22.8, 30.5, 55.7, 66.2, 97.3, 186.5, 240.0.
\]

Are the above values from a normal or a lognormal distribution?
Solution. A normal quantile plot of the observed values follows. Clearly, the variable $x$, the hourly median power, is not normally distributed, as the normal quantile plot is curvilinear.
(b) By taking the natural logarithm of the variable and constructing a normal quantile plot of the transformed data, we obtain:

This plot looks quite linear indicating that it is plausible that these observations were sampled from a lognormal distribution.
3. Weibull Quantile Plot.

If $X \sim W(\alpha, \beta)$, then area to the left of $\eta_p$, the $p$-th quantile, satisfies

$$1 - e^{-\left(\frac{\eta_p}{\beta}\right)^\alpha} = p$$
$$\Rightarrow e^{-\left(\frac{\eta_p}{\beta}\right)^\alpha} = (1 - p)$$
$$\Rightarrow \left(\frac{\eta_p}{\beta}\right)^\alpha = -\ln(1 - p).$$

Taking log again we get,

$$\alpha[\ln(\eta_p) - \ln(\beta)] = \ln[-\ln(1 - p)]$$
$$\Rightarrow \alpha\ln(\eta_p) + \gamma = \ln[-\ln(1 - p)].$$

Therefore,

$$\ln(-\ln(1 - p)) = \alpha\ln(\eta_p) + \gamma$$
$$\Rightarrow \ln(-\ln(1 - p)) \text{ and } \ln(\eta_p) \text{ are linearly related.}$$
Let \( p_i = \frac{i-0.5}{n} \), \( 1 \leq i \leq n \). Then if data \( x_1, \ldots, x_n \) are from Weibull, then \( \ln(\eta_{p_i}) \) is approximately equal to \( \ln(x_{(i)}) \).

Hence, \( \ln[-\ln(1 - p_i)] \) and \( \ln(x_{(i)}) \) are linearly related.

Plot the points \( (\ln(-\ln(1 - p_i)), \ln(x_{(i)})), \ 1 \leq i \leq n \), where \( p_i = \frac{i-0.5}{n} \).

If the graph is linear (not necessarily the line passing through the origin), then \( x_1, \ldots, x_n \) are from \( W(\alpha, \beta) \) for some \( \alpha \) and \( \beta \).

Note that we do not have to compute the population quantiles \( \eta_{p_i} \).
Example 13 The following are modulus of elasticity observations for cylinders.

37.0, 37.5, 38.1, 40.0, 40.2, 40.8, 41.0,
42.0, 43.1, 43.9, 44.1, 44.6, 45.0, 46.1, 47.0,
62.0, 64.3, 68.8, 70.1, 74.5.

Use the quantiles for a sample of size 20 given above to construct a normal quantile plot, and comment on the plausibility of a normal population distribution.
Solution:
The normal quantiles are easy to generate using Matlab. The Quantile plot is:

The graph shows some nonlinearity and hence the data may not come from a normal distribution.
Example 14
The following observations are on lifetime (hr) of power apparatus insulation when thermal and electric stress acceleration were fixed at particular values:

\[ 282, 501, 741, 851, 1072, 1122, 1202, 1585, 1905, 2138 \]

Construct a Weibull plot and comment.
Solution:
This Weibull quantile plot was created in Minitab. The graph is given below:

The plot does not show any marked deviation from linearity. So, a Weibull distribution would be plausible for this data.
Continuous Distributions and Probability Plots

Homework

Sect 4.4: 61, 66, 70
Sect 4.5: 72, 76, 81, 86
Sect 4.6: 88, 92