Nonlocal Diffusion Models as Bridges Between Local and Fractional Diffusion Models

Abstract

We present a mathematical framework of nonlocal diffusion models that are characterized by horizon parameters measuring the ranges of nonlocal interactions and memory effects. We establish a number of mathematical properties of the nonlocal models and also explore their various limits. In particular, we show how they can effectively approximate the classical diffusion in the local limit when horizon parameters shrinking to zero and the fractional diffusion in the global limit when horizon parameters tending to infinity. We also discuss seamless coupling of local and nonlocal models.