MICHIGAN STATE UNIVERSITY
Department of Statistics and Probability

A Workshop on Future Directions in Fractional Calculus Research and Applications

Weihua Deng
Lanzhou University

Mean exit time and escape probability for the anomalous processes with the tempered power-law waiting times

Abstract

The mean first exit (passage) time characterizes the time of a stochastic process never leaving a fixed region in the state space, while the escape probability describes the likelihood of a transition from one region to another for a stochastic system driven by discontinuous (with jumps) Levy motion. This paper discusses the two deterministic quantities for the anomalous processes having the tempered Levy stable waiting times with the tempering index $\lambda>0$ and the stability index $0<\alpha \leq 1$; as for the distribution of jump lengths or the type of the noises driving the system, two cases are considered, i.e., Gaussian white noises and non-Gaussian β-stable ($0<\beta<2$) Levy noise. Firstly, we derive the nonlocal elliptic partial differential equations (PDEs) governing the mean first exit time and escape probability. Based on the derived PDEs, further investigations are performed. It is observed that the mean first exit time depends strongly on the domain size and the values of α, β and λ; when λ is close to zero, the mean first exit time tends to ∞. In particular, we also find an interesting result that the escape probability of a particle with power-law jumping length distribution is independent of the distribution of waiting times.

For the solutions of the derived PDEs, the boundary layer phenomena is observed, which inspires the motivation for developing the boundary layer theory for nonlocal PDEs. The connections between the derived PDEs and the tempered fractional Feynman-Kac equations, and the probability distributions of the mean exit time should be the coming interesting research topics. The rigorous theoretical proof for the well-posedness of the derived PDEs is an open problem.

This is a joint work with Xiaochao Wu.