UCMERCED MESA LAB

Regional Sensing & Actuation of Fractional Order Distributed Parameter Systems

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October 21, 2016. Friday. 11am-12noon

A Workshop on Future Directions in Fractional Calculus Research and Applications, MSU, East Lansing, MI.

Thanks

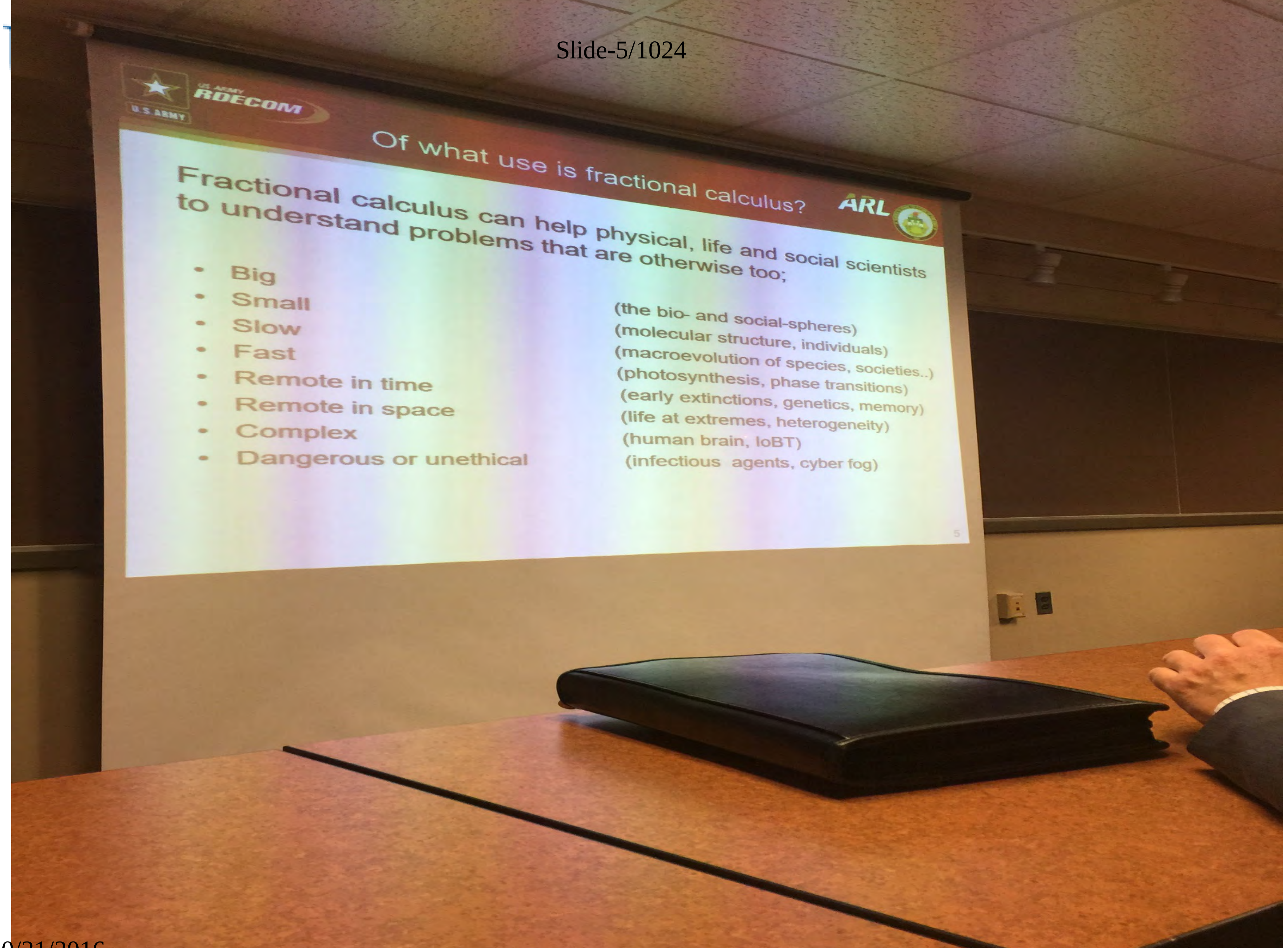
- Mark! Congratulations!
- All invited lecturers! I learned a lot!
- You all for staying.
- Colors on MSU campus!

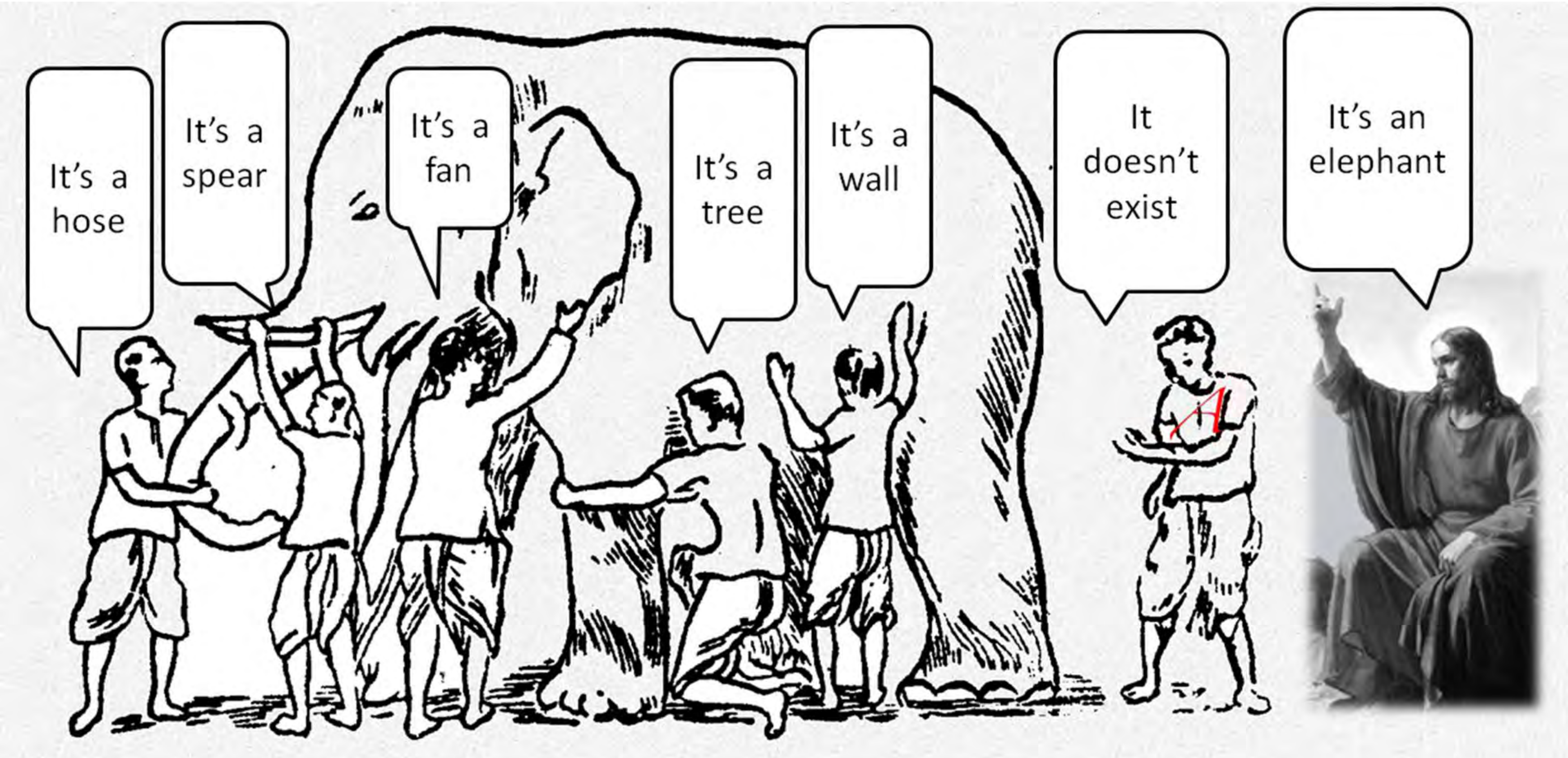
Two questions

- So what? / Why bother?
- What else I/you/we can do?

So what? / Why bother?

- Three answers
  - Complexity
  - Better than the best
  - xxx


U.S. ARMY
RDECOM
Of what use is fractional calculus?
ARL
Fractional calculus can help physical, life and social scientists to understand problems that are otherwise too;
Big (the bio- and social-spheres)
Small (molecular structure, individuals)
Slow (macroevolution of species, societies..)
Fast (photosynthesis, phase transitions)
Remote in time (early extinctions, genetics, memory)
Remote in space (life at extremes, heterogeneity)
Complex (human brain, IoBT)
Dangerous or unethical (infectious agents, cyber fog)



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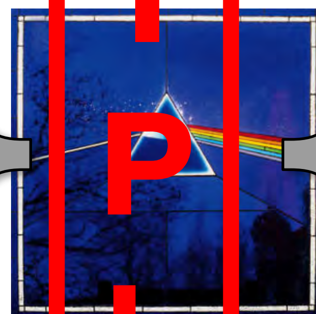
Complex systems. phenomena, behaviors, …



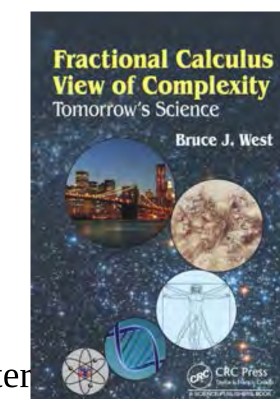
I
P
L

Scale-Free,
Heavy-Tailedness,
Long Range Dependence,
Long Memory …

Fractional order dynamic

Fractional calculus based models

Mittag-Leffler Law



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New wisdom equipped with FC

- 道可道，非常道。------世间万物的运行规律是可以被描述的，但它们并非一成不变的。

Non-normal way:
Fractional Calculus!
Heavytailedness

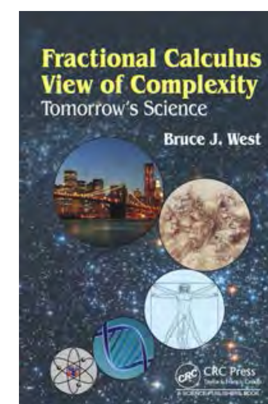
way

by non-normal

be explained

could

The nature’s rule (of complexity)



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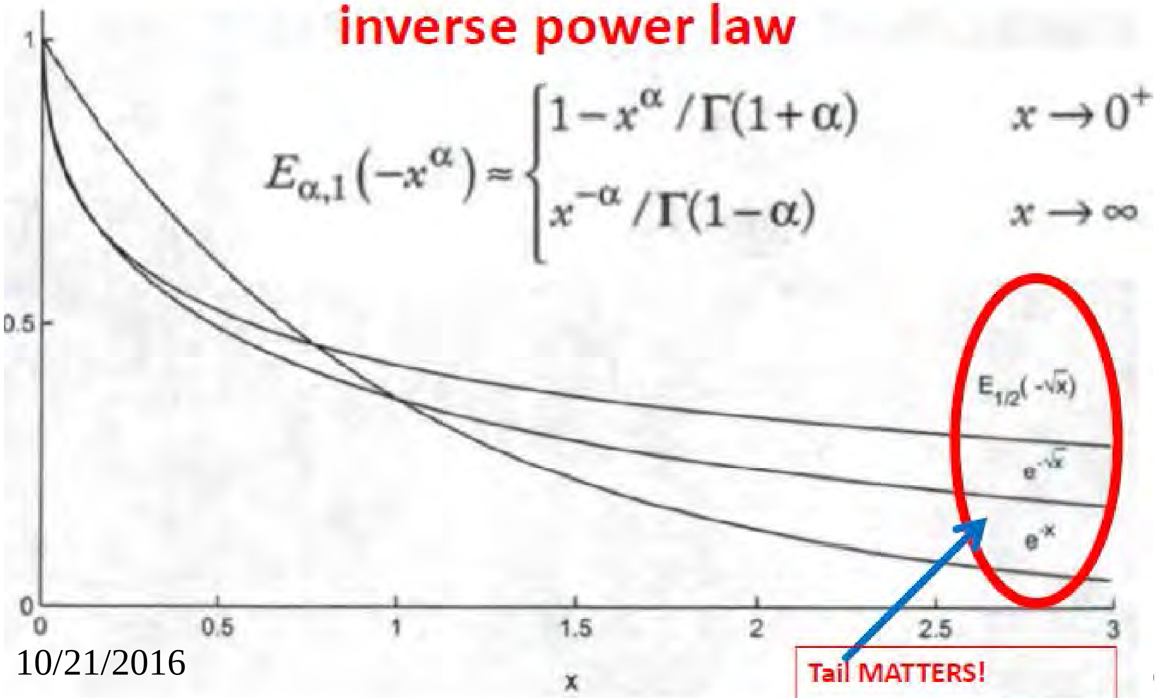
New wisdom equipped with FC

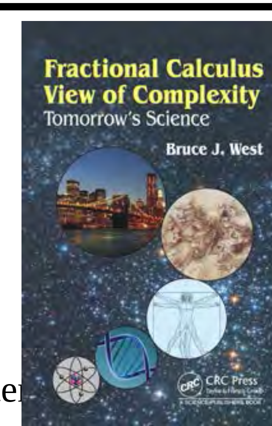
- 玄之又玄，众妙之门。-------了解这类对立统一体相互转变的规律，就是通向对世间万物理解的大门。

Root of long (algebraic) tail, or inverse power law



Non-normal way:

Fractional Calculus! Heavytailedness



New wisdom equipped with FC

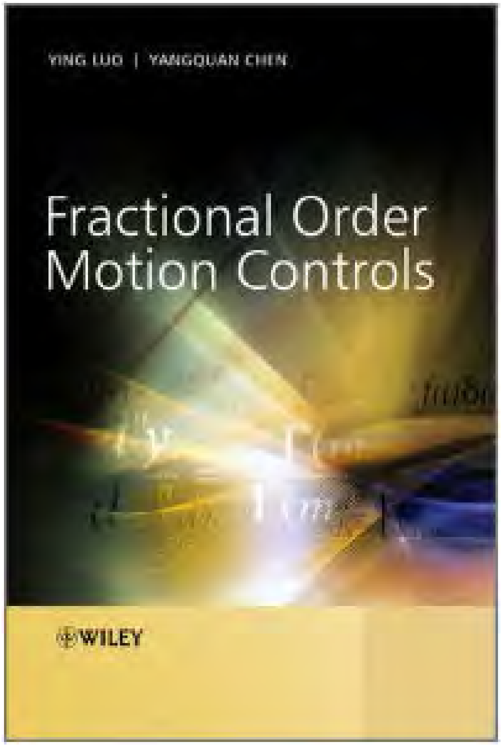
- **"God is in the detail"**
- **"The Devil is in the detail"**
  - http://en.wikipedia.org/wiki/The_Devil_is_in_the_detail

- **"God is in the tail"**
- **"The Devil is in the tail"**

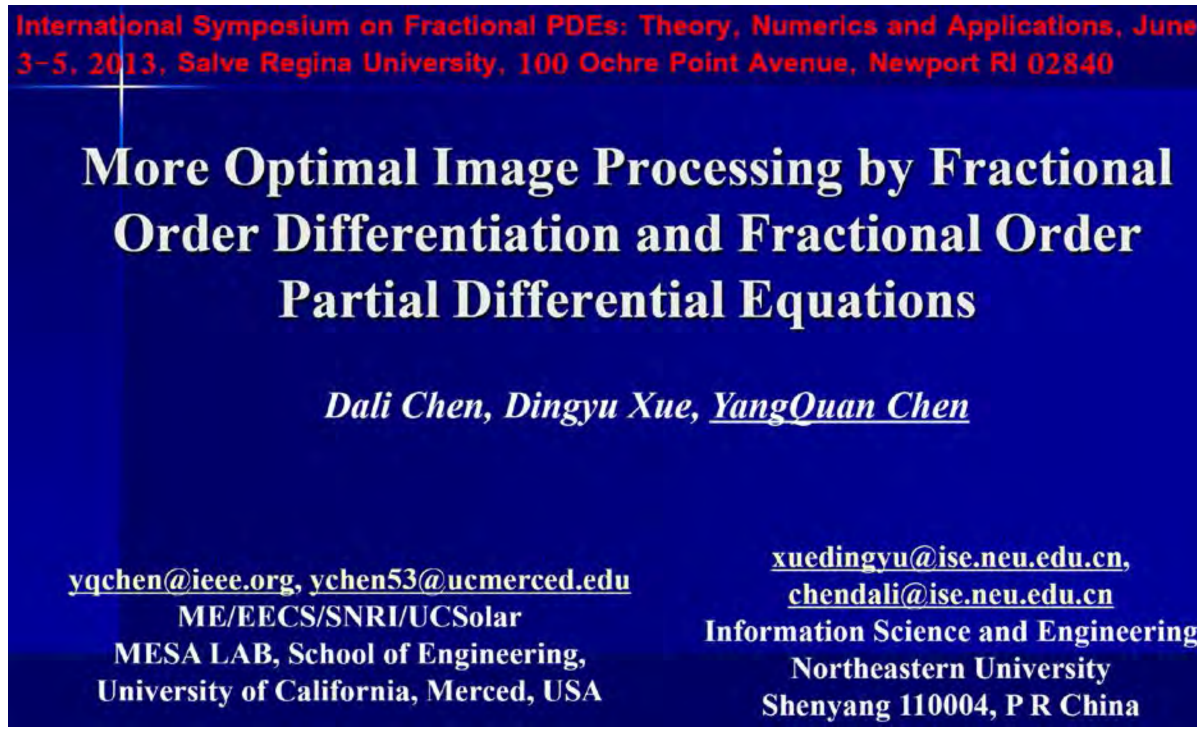
So what? / Why bother?

- Three answers
  - Complexity
  - **Better than the best**
  - xxx

Better than the best, “more optimal”



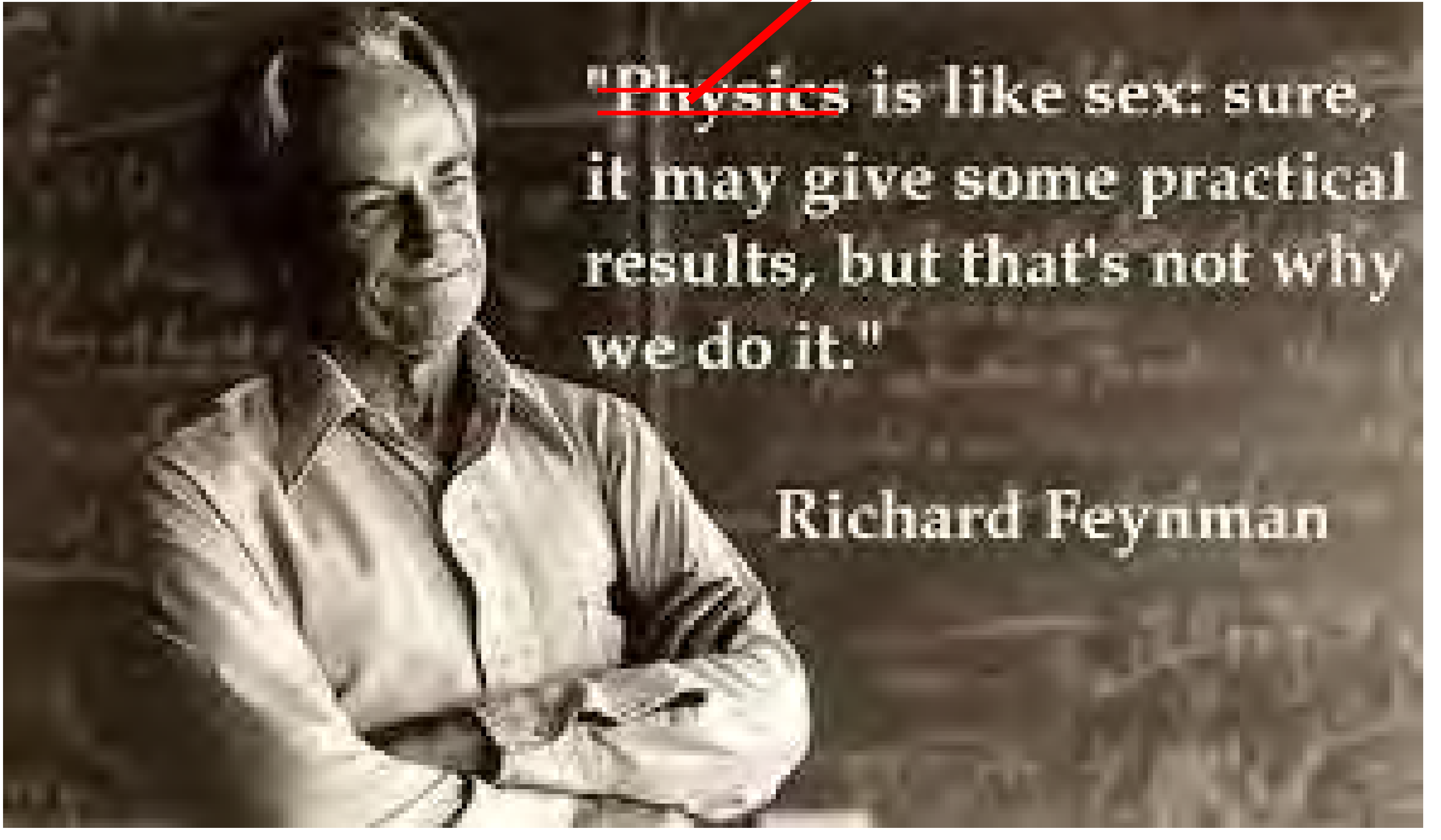
2012



So what? / Why bother?

- Three answers
  - Complexity
  - Better than the best
  - **xxx**

FC


"Physics is like sex: sure,
it may give some practical
results, but that's not why
we do it."
Richard Feynman

Outline (What else I can do?)

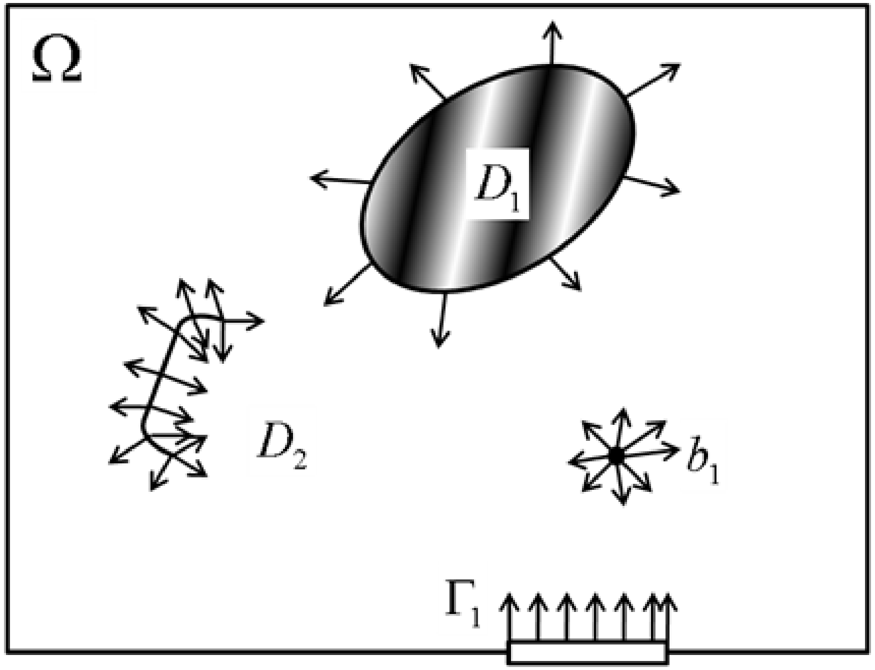
- **Motivations**
- **Why Regional?**
- **Some recent results**
- **Future topics**

What makes “measurement” so hard?

- For distributed parameter systems (DPS), the system dynamics evolves along time as well as spatial variables, usually governed by partial differential equations (PDEs) and usually it is required to discuss the modeling and control problems in terms of a specified domain.
- Additionally, for sensing and actuation, we also need to consider the cases of effective domains such as point-wise sensing/actuation, zone distributed sensing/actuation, and whole domain distributed sensing/actuation. Additionally, we can also consider sensors/actuator can be movable within the domain of interest.

Measurement – It is no longer as simple and straightforward as before any more! WWWWWH issues

- Why measure?
- What to measure?
- Who to measure
- Where to measure?
- When to measure?
- How to measure?



2x2x5x5 = 100 cases

	sensors	actuator
Mobile or static ?	Point-wise	Point-wise
	zone	zone
	Domain distributed	Domain distributed
	filament	filament
	Boundary	Boundary

x 2

x2: sensors and actuators are collocated or noncollocated

x 4

x4: communicating (sensor-to-sensor, actuator-to-actuator, sensor-to actuator, actuator-to-sensor).

Note: scanning an array of sensors/actuators is considered as "mobile"!

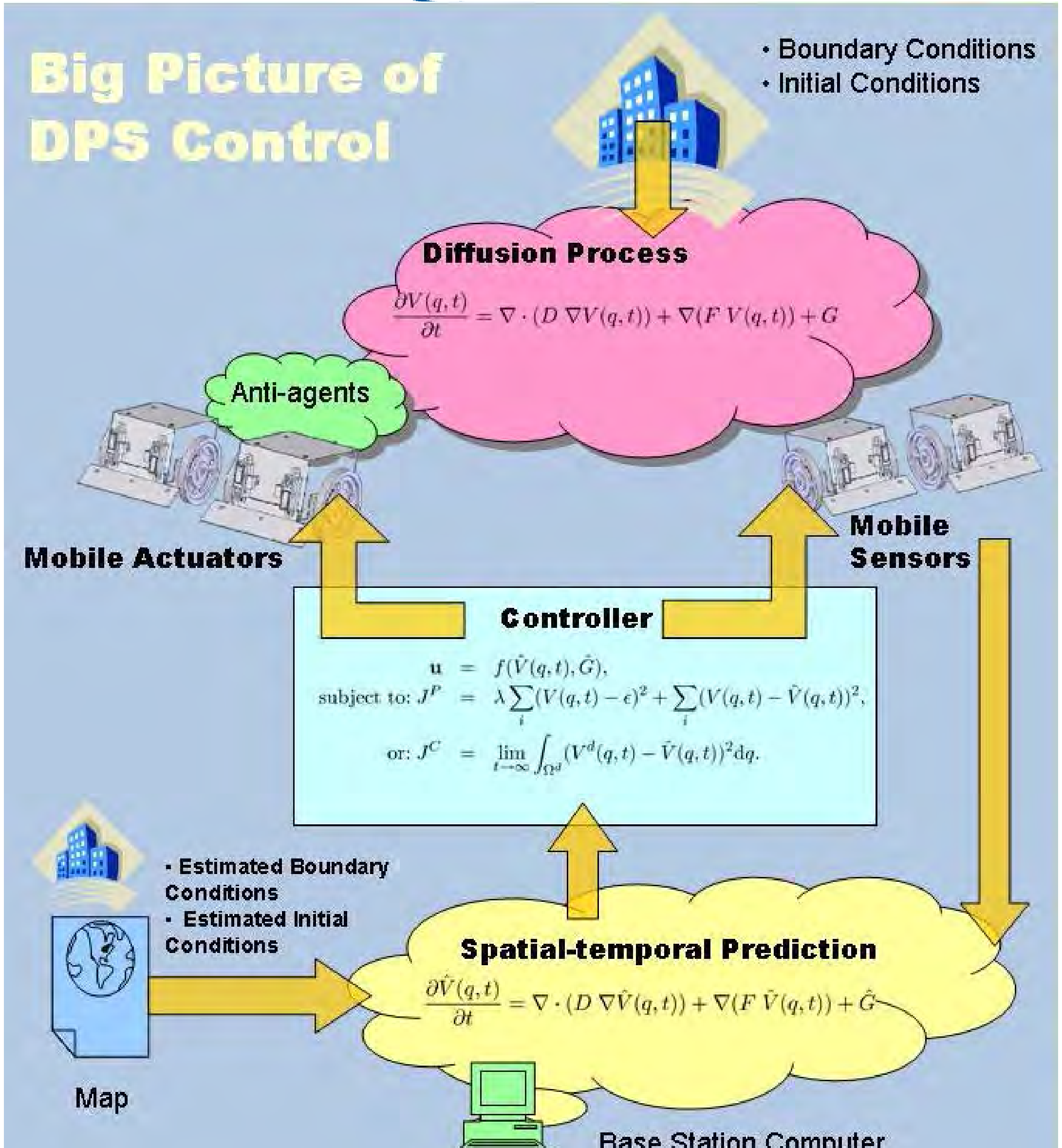
So, total cases: 2x2x5x5x2x4=800.

(**Ph.D.s**)

Example-1

For a simple room heater control system, we have

- one static point-wise sensor (thermostat)
- one static zonal actuator (heater).
- In this case, sensor and actuator are placed at different places, also known as "non-collocated".



DPS:
distributed parameter systems

Features:

- Domain of interest
- Sensor configuration
- Sensor effective region
- Actuator configuration
- Actuator effective region
- Mobile or static
- Communicating or not
- Collocated or not

MAS-net Project:

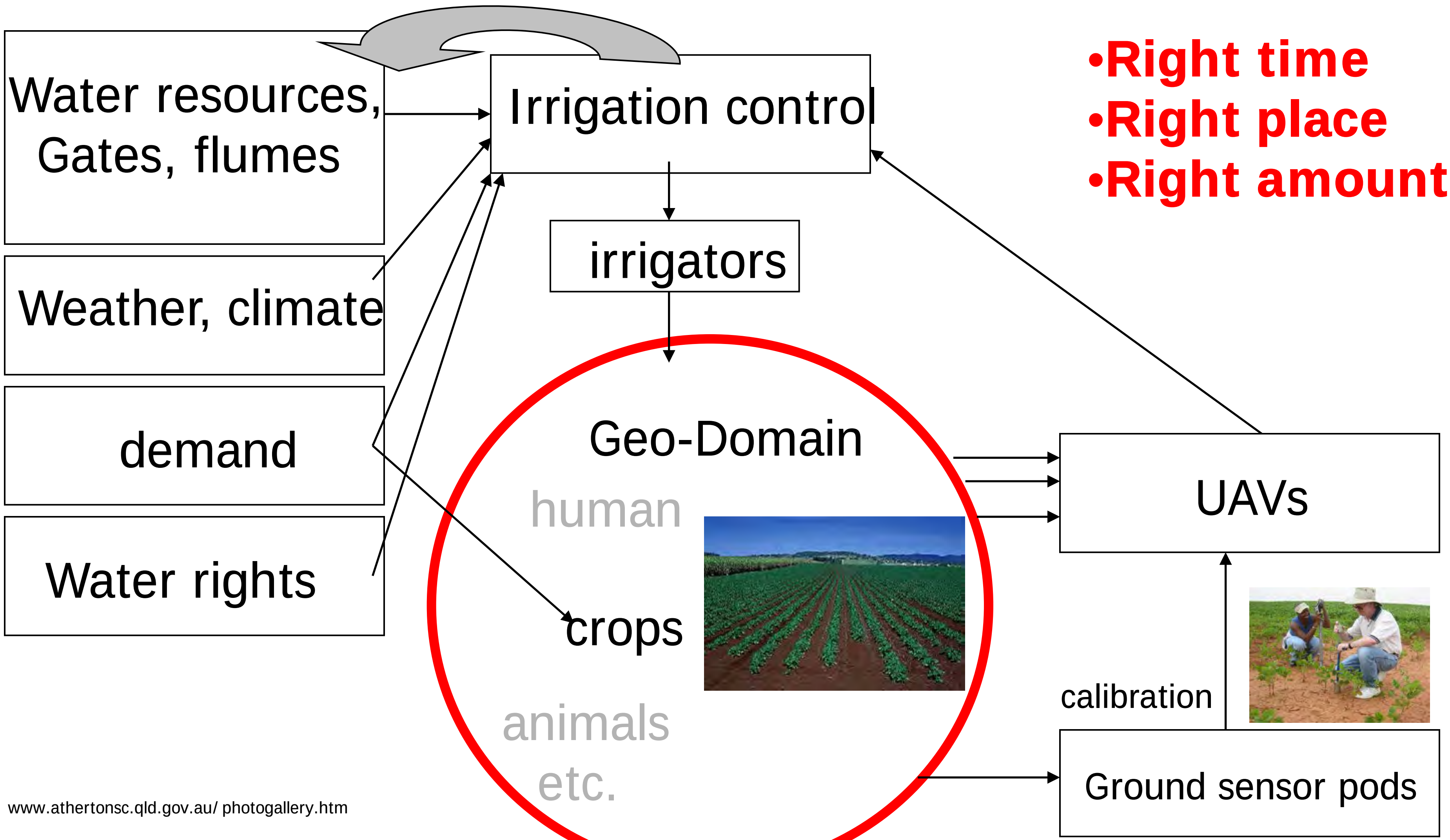
Smart Sniffing and Spraying Problem

Sensors and actuators are all mobile

Credit: Drs. YangQuan Chen and Kevin L. Moore, 2002

WaterWatch?

- **Right time**
- **Right place**
- **Right amount**



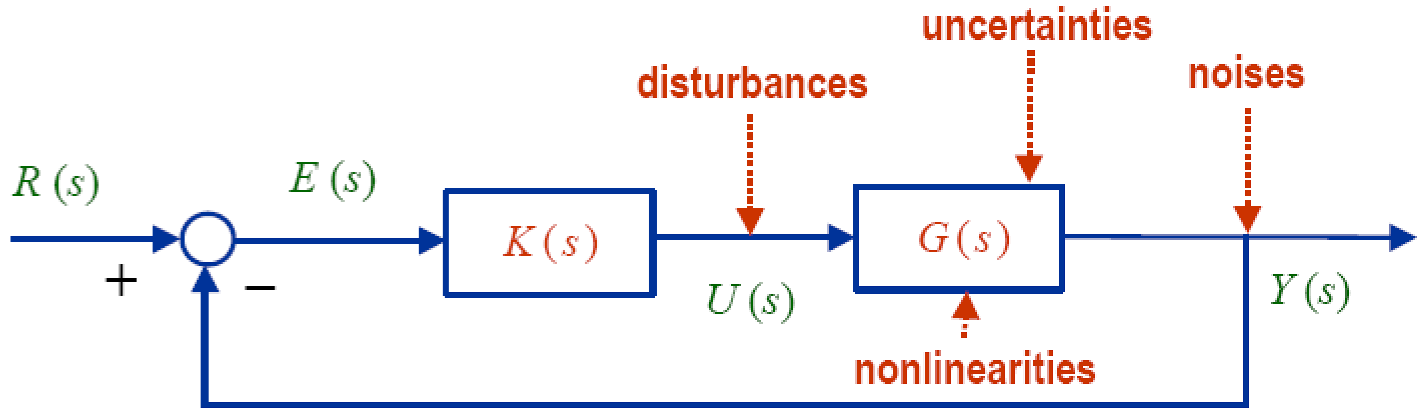
www.athertonsc.qld.gov.au/ photogallery.htm

Credit: Dr. YangQuan Chen, 2005

http://www.nasa.gov/centers/goddard/news/topstory/2004/0729soilshowdown.html

Fractional Order Controls

- **IO Controller + IO Plant**
- **FO Controller + IO Plant**
- **FO Controller + FO Plant**
- **IO Controller + FO Plant**



D. Xue and **Y. Chen***, "**A Comparative Introduction of Four Fractional Order Controllers**". Proc. of The 4th IEEE World Congress on Intelligent Control and Automation (WCICA02), June 10-14, 2002, Shanghai, China. pp. 3228-3235.

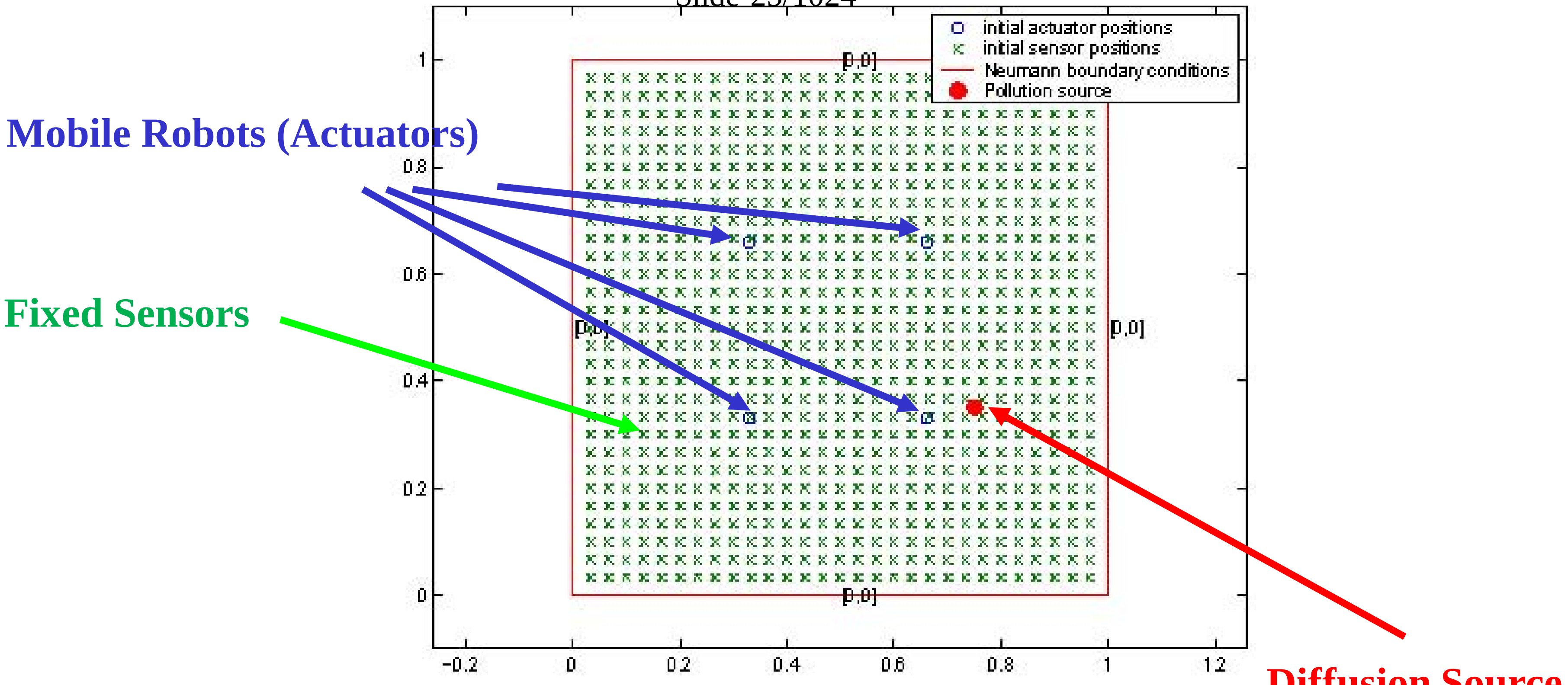
x 4

- Process is integer order or fractional order
- Controller is integer order or fractional order

x 4

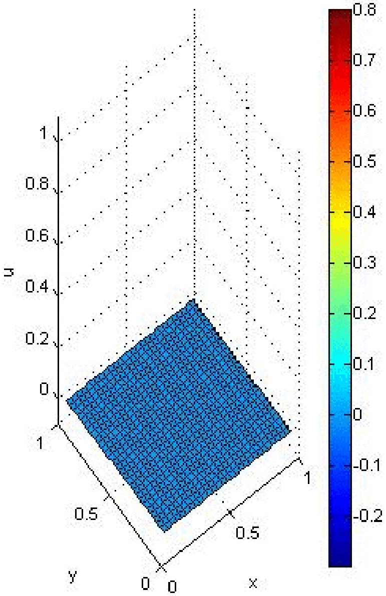
- Controller is regional or not
- Observer is regional or not

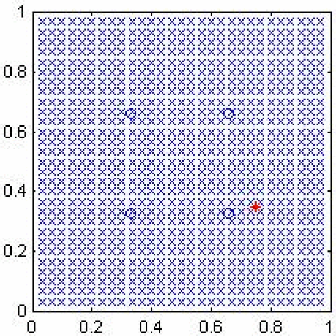
800x16 = 12,800 cases (PhDs)



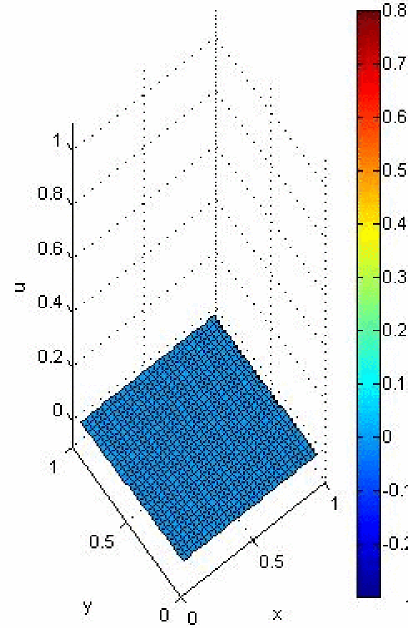
Initial layout of actuators and sensors.

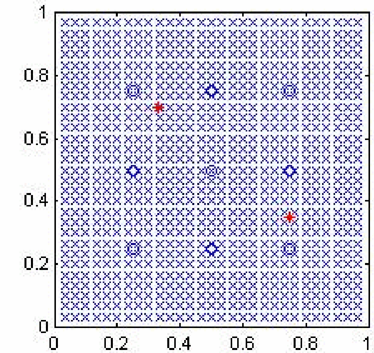
Strategy: **1) Form Voronoi tessellation**
2) Move each robot to the mass centroid of its region
3) Spray neutralizing chemical in amount proportional to concentration in region



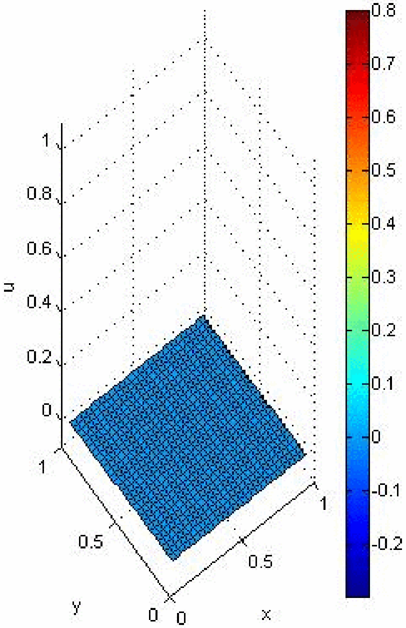


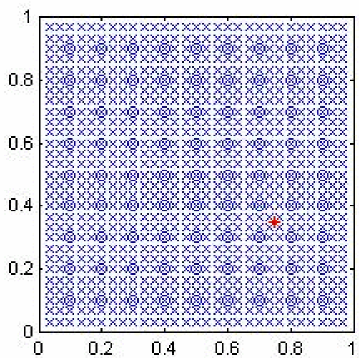
4 robots sprayers, one contaminant source



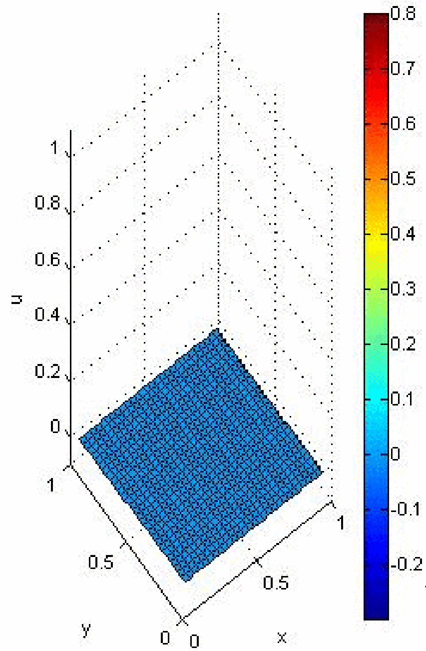


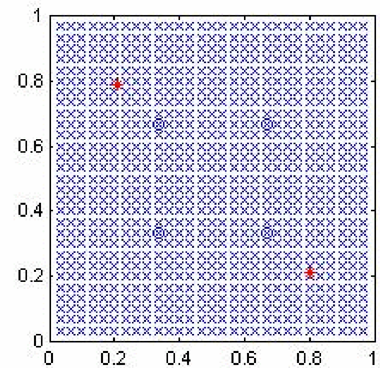
9 robots sprayers, two contaminant sources



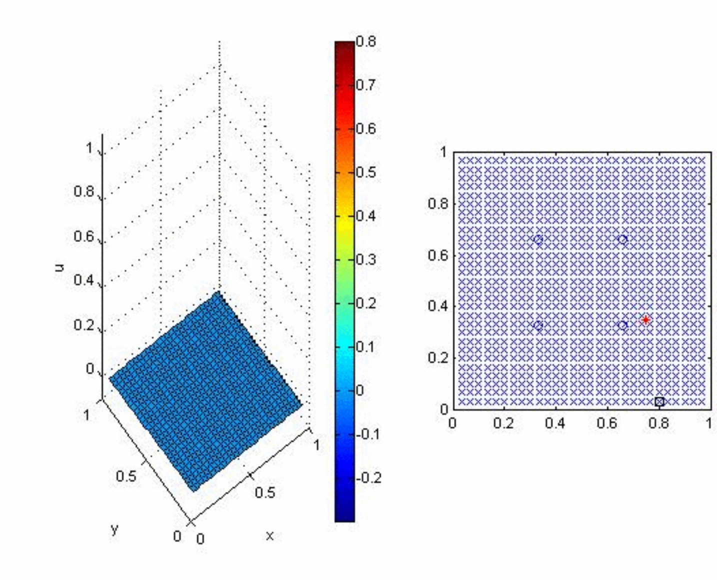


81 robots sprayers, one contaminant source



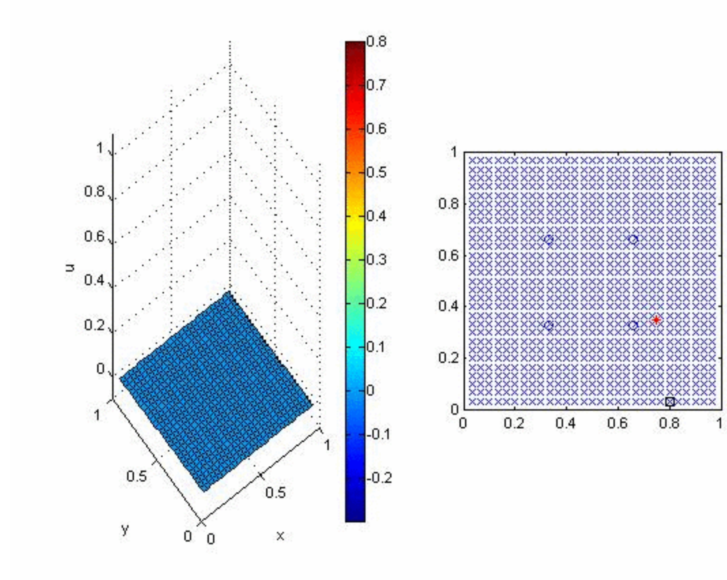


4 robots sprayers, two contaminant sources (moving)



4 robots sprayers,
one contaminant source, moving obstacle.

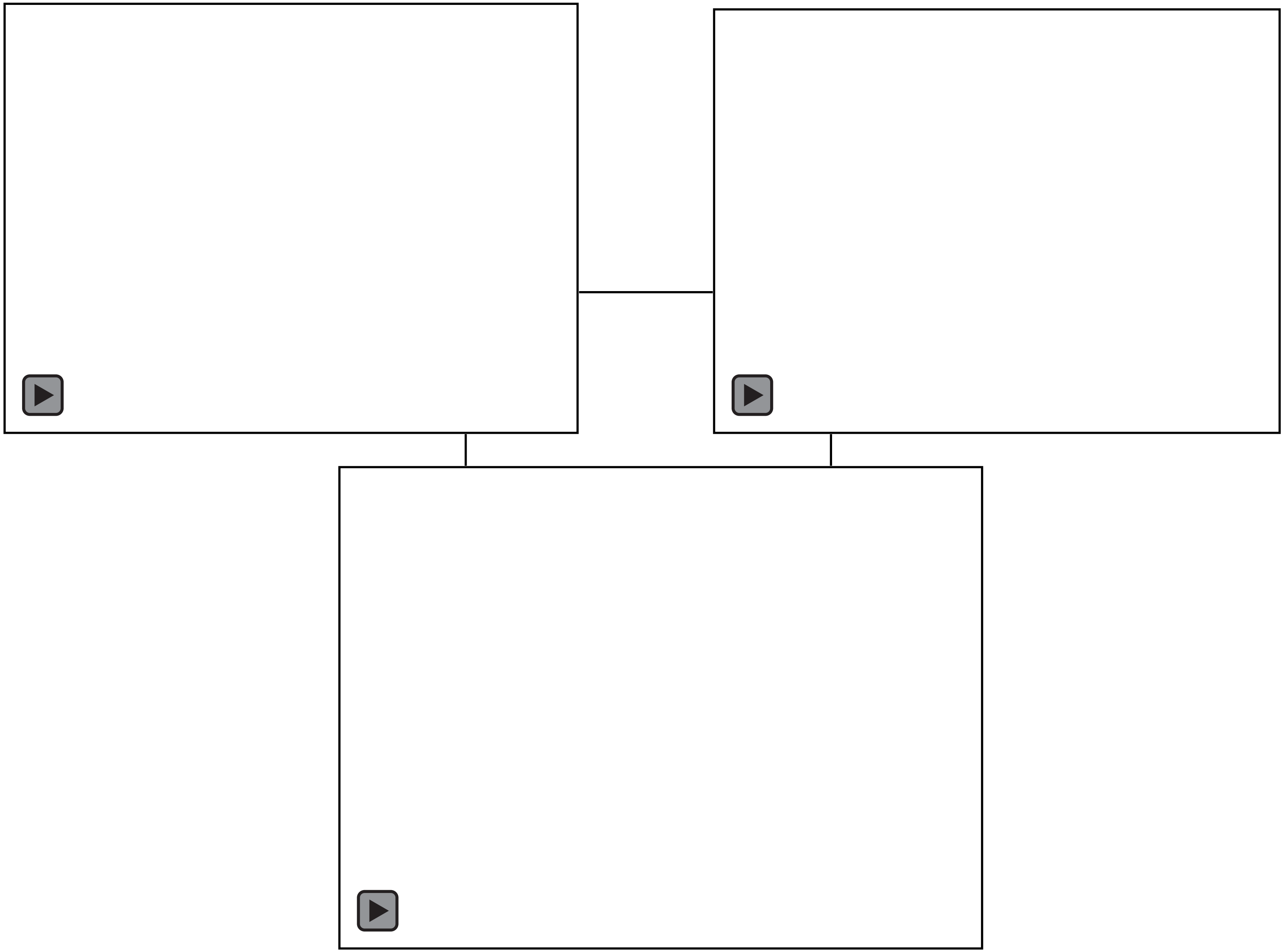
Normal potential field.



4 robots sprayers,
one contaminant source, moving obstacle.

Fractional order potential field.

Can specify the "degree of danger" of the obstacle in potential field method



My submission - "Computational" can be put in front of almost every thing

- – Computational intelligence
- – Computational material
- – Computational neuron science
- – Computational psychology
- – Computational fluid dynamic
- – Computational biology
- – Computational chemistry
- – Computational ecology
- – Computational social science
- – Computational virology
- – ….

My submission - "Control" can be put after almost every thing

- Speed Control
- Diet Control
- Weight Control
- Emotion Control
- Arm Control
- Microclimate Control
- Machine Control
- Human Gait Control
- Blood-pressure Control
- Aging Control
- Evacuation Control/Traffic Control/Conggestion Control
- ….

So, here comes CPS

Computational Thinking + Control Thinking

=> Cyber Physical Systems

Dr. Chen's Definition of CPS:
Computational thinking and integration of computation around the physical dynamic systems form the Cyber-Physical Systems (CPS) where sensing, decision, actuation, computation, networking, and physical processes are mixed.

Common attributes of CPS's

- Computational thinking, not just "computational doing"
- Integration of computation around the physical dynamic systems
- Sensing, decision, actuation, computation, networking, and physical processes are mixed.
- Bigger **closed-loop** picture
- **Mostly infinite-dimensional spatial-temporal complex dynamic systems.**

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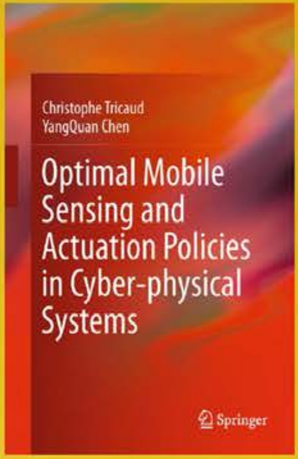
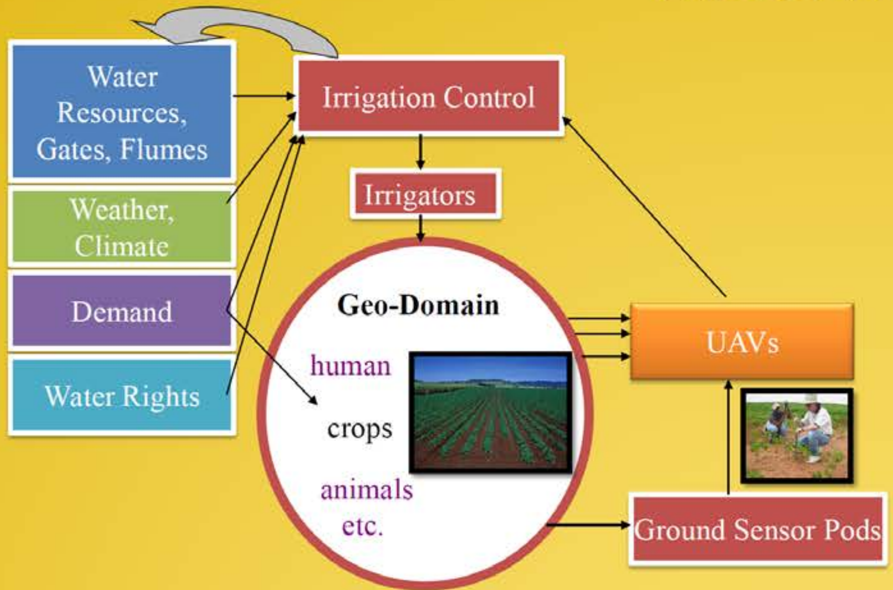
Cyber-Physical Systems

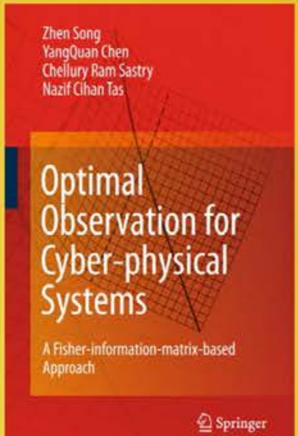


WaterWatch?



More Real-time Water Management

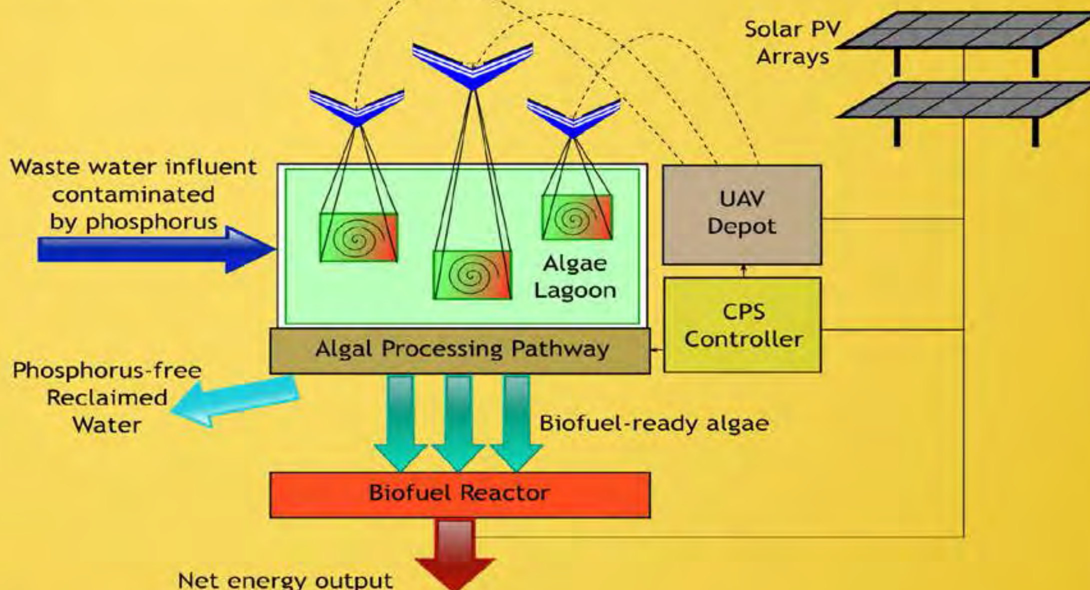
- •At the right time
- •At the right place
- •With the right amount

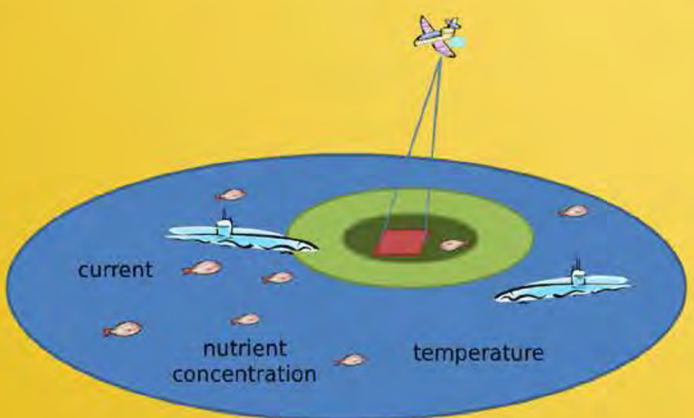


Credit: Dr. YangQuan Chen, 2005

Water Treatment Lagoon

- UAS-enabled CPS to create biofuel from waste
  - UASs provide accurate remote sensing data

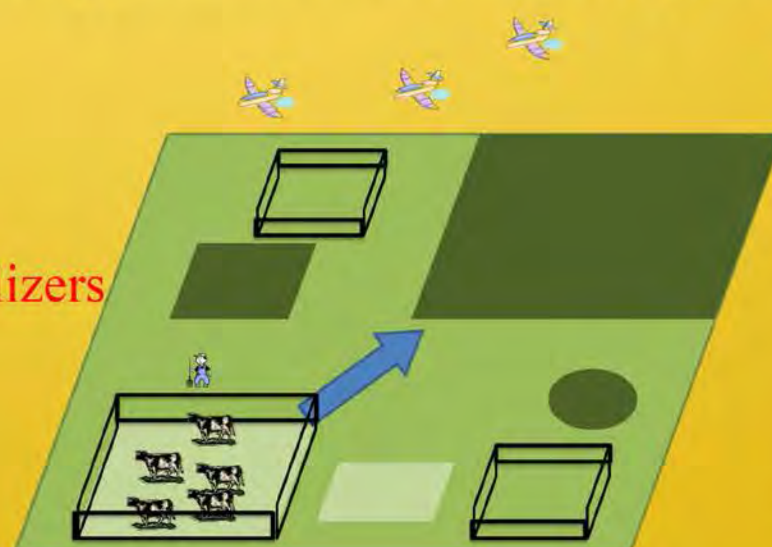


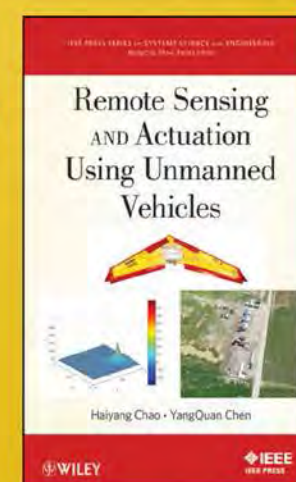


Algal Bloom Tracking & Modeling

Agriculture in 2100s

- when no more synthetic fertilizers available





Outline (What else I can do?)

- **Motivations**
- **Why Regional?**
- **Some recent results**
- **Future topics**

"Physical": Distributed Parameter Systems (DPSs)

DPSs are systems where the parameters and the variables depend both on time and the location. They include inputs and outputs which allow the system to communicate and interact with its environment (controls and measurements).

For example:

$$y_t(x,t) = -\triangle y(x,t) + u(x,t) \text{ in } \Omega \times [0,1]$$

with initial and boundary conditions:

$$y(\eta,t) = 0 \text{ on } \partial\Omega \times [0,1], \quad y(x,0) = y_0(x) \in L^2(\Omega),$$

where u is a control depends on the number and the structure of actuators.

The measurements $C : L^2(\Omega \times [0,1]) \to Z$ is given by $z(x,t) = Cy(x,t)$ according to the number and the structure of sensors.

Difficulties for classical investigation of CPSs

(1) For DPSs, the system evolves along time as well as spatial variables, usually governed by partial differential equations (PDEs).

(2) For sensors: It is not easy to make enough measurements--neither dangerously enough nor luxuriously enough.

(3) For actuators: It is not easy and not a good idea to control whole system. (waste time and cost consuming)

(4) Moreover, in the case of diffusion systems, in general, not all the states can be reached in the whole domain of interest.

Benefits of regional control and observation

(a) Occurs naturally;

(b) Allow for a reduction in the number of physical actuators/sensors ;

(c) Help to reduce the computational requirements;

(d) Great help to study those non-controllable/non-observable system since it interests in knowledge of the states only in a critical sub-region of the system domain;

(e) To improve the degree of controllability/observability of the system only on a sub-region;

... .

Regional controllability

Let Ω be an open bounded subset of $\mathbf{R}^n$ with smooth boundary $\partial\Omega$ and consider the following Riemann-Liouville type time fractional diffusion systems

$$\begin{cases} {}_0D_t^{\alpha} z(t) + Az(t) = Bu(t), \quad t \in [0, b], \ 0 < \alpha < 1, \\ \lim\limits_{t \to 0^+} {}_0I_t^{1-\alpha} z(t) = z_0 \in L^2(\Omega), \end{cases} \tag{1}$$

$$z(t) = t^{\alpha-1} K_\alpha(t) z_0 + \int_0^t (t-s)^{\alpha-1} K_\alpha(t-s) Bu(s) ds, \tag{2}$$

Let $\omega \subseteq \Omega$ be a given region of positive Lebesgue measure and $z_b \in L^2(\omega)$(the target function) be a given element. Consider now the restriction map

$$p_\omega : L^2(\Omega) \to L^2(\omega), \tag{3}$$

defined by $p_\omega z = z|_\omega$, is the projection operator on ω.

Definition 1

The system (1) is said to be regionally exactly (approximately) controllable in ω at time b if for any $z_b \in L^2(\omega)$, given $\varepsilon > 0$, there exists a control $u \in U$ such that

$$p_\omega z(b, u) = z_b \left(\| p_\omega z(b, u) - z_b \| \leq \varepsilon \right). \tag{4}$$

Lemma 3.1.1. *For any given* $f \in L^2(0,b;Z)$, $0 < \alpha < 1$, *a function* $v \in L^2(0,b;Z)$ *is said to be a mild solution of the following system*

$$\begin{cases} {}_0D_t^{\alpha} v(t) + Av(t) = f(t),\ t \in [0,b], \\ \lim_{t\to 0^+} {}_0I_t^{1-\alpha} v(t) = v_0 \in Z, \end{cases}$$

if it satisfies $v(t) = t^{\alpha-1}K_\alpha(t)v_0 + \int_0^t (t-s)^{\alpha-1}K_\alpha(t-s)f(s)ds$, *where*

$$K_\alpha(t) = \alpha \int_0^\infty \theta \phi_\alpha(\theta)\Phi(t^\alpha \theta)d\theta.$$

Here $\{\Phi(t)\}_{t\geq 0}$ *is the strongly continuous semigroup generated by operator* $-A$, $\phi_\alpha(\theta) = \frac{1}{\alpha}\theta^{-1-\frac{1}{\alpha}}\psi_\alpha(\theta^{-\frac{1}{\alpha}})$ *and* ψ_α *is a probability density function defined by* $\psi_\alpha(\theta) = \frac{1}{\pi}\sum_{n=1}^{\infty}(-1)^{n-1}\theta^{-\alpha n-1}\frac{\Gamma(n\alpha+1)}{n!}\sin(n\pi\alpha)$, $\theta > 0$ *such that*

$$\int_0^\infty \psi_\alpha(\theta)d\theta = 1 \text{ and } \int_0^\infty \theta^\nu \phi_\alpha(\theta)d\theta = \frac{\Gamma(1+\nu)}{\Gamma(1+\alpha\nu)},\ \nu \geq 0.$$

Proof. Omitted.

Zhou Y, Jiao F (2010) Existence of mild solutions for fractional neutral evolution equations. Comput Math Appl 59(3):1063–1077

Consider the following attainable set $D(t)$ in $L^2(\Omega)$:

$$D(t) = \left\{ d(t, \cdot) \in L^2(\Omega) : d(t, x) = \int_0^t (t-s)^{\alpha-1} K_\alpha(t-s) Bu(s) ds \right\}$$

and we have

Lemma 2

For any given $b > 0$, the necessary and sufficient condition for the exact (approximate) controllability of the system (1) at time b is that

$$D(b) = L^2(\Omega) \left(\text{respectively, } \overline{D(b)} = L^2(\Omega) \right),$$

where $D(t)$ $(t > 0)$ is a linear manifold and $\overline{D(b)}$ is the closure of $D(b)$.

By Lemma 2, it suffices to suppose that $z_0 = 0$ in the following discussion.

Why regional?

Consider the following example, which is not controllable on the whole domain but regionally controllable on ω.

$$\begin{cases} {}_0D_t^{\alpha} z(x,t) - \frac{\partial^2}{\partial x^2} z(x,t) = p_{[a_1,a_2]} u(t) \quad \text{in } [0,1]\times[0,b], \\ \lim_{t\to 0^+} {}_0I_t^{1-\alpha} z(x,t) = z_0(x) \quad \text{in } [0,1], \\ z(0,t) = z(1,t) = 0 \quad \text{in } [0,b], \end{cases} \tag{5}$$

where $0<\alpha<1$, $Bu = p_{[a_1,a_2]}u$ and $0 \le a_1 \le a_2 \le 1$. Moreover, since $A = -\frac{\partial^2}{\partial x^2}$, from [Pazy(2012)], we get that $\lambda_i = i^2\pi^2$, $\xi_i(x) = \sqrt{2}\sin(i\pi x)$ are respectively, the eigenvalue and eigenfunction of A. What's more, we have

$$(H^* z)(t) = (b-t)^{\alpha-1} \sum_{i=1}^{\infty} E_{\alpha,\alpha}(-\lambda_i (b-t)^{\alpha})(z,\xi_i)_{L^2(0,1)} \int_{a_1}^{a_2} \xi_i(x)dx.$$

By $\int_{a_1}^{a_2} \xi_i(x)dx = \frac{\sqrt{2}}{i\pi} \sin\frac{i\pi(a_1+a_2)}{2} \sin\frac{i\pi(a_2-a_1)}{2}$, we get that $\overline{Im(H)} \neq L^2(\omega)$ when $a_2 - a_1 \in \mathbf{Q}$. Then the system (5) is not controllable on $[0,1]$.

Next, we show that there exists a sub-region $\omega \subseteq \Omega$ such that the system can be regionally controllable in ω at time b.

Without loss of generality, let $a_1 = 0$, $a_2 = 1/2$, $z_* = \xi_k, (k = 4j, j = 1,2,3,\cdots)$. Based on the argument above, z_* is not reachable on $\Omega = [0,1]$. However, let $\omega = [1/4, 3/4]$, we see that

$$(H^* p_\omega^* p_\omega z_*)(t) = \sum_{i \neq 4j} \frac{\sqrt{2} E_{\alpha,\alpha}(-\lambda_i (b-t)^\alpha)}{i\pi (b-t)^{1-\alpha}} \int_{1/4}^{3/4} \xi_i(x) \xi_{4j}(x) dx \left[1 - \cos(i\pi/2)\right]$$
$$\neq 0.$$

Then z_* can be regionally controllable in $\omega = [1/4, 3/4]$ at time b.

Outline (What else I can do?)

- **Motivations**
- **Why Regional?**
- **Some recent results**
- **Future topics**

http://perso.univ-perp.fr/aej/index.html

Université de Perpignan

Professor Abdelhaq El Jai's works

- ftp://169.236.9.29/El-Jai-collection/
- **L. Afifi, A.El Jai et E. Zerrik.** Systems Theory. Regional Analysis of Infinite Dimensional Linear Systems, PUP, ISBN : 978-2-35412-140-2, 440 pages. Janvier 2012
- **A. El Jaï & A.J Pritchard.** *Sensors and Controls in the Analysis of Distributed Systems*. Ellis Horwood Ltd - JOHN WILEY & SONS 1988

Why we made this collection of El Jai's works?

- Professor Abdelhaq EL JAI's papers on regional analysis of distributed parameter systems (DPS) are truly original and important. It is getting more and more important in this IoT (internet of things) and CPS (cyber-physical systems) age with cloud computing and big data movements. A dedicated conference was organized in his honor in 2014: http://cmacs2014.online.fr/
- My first reading of El Jai's work was in 2002. In my 2004 SPIE paper on mobile actuator and sensor networks (MAS-net), we first cited El Jai's work.
- Since 2014, with Fudong Ge, a visiting Ph.D. student from Donghua University, China, we started an exciting journey in exploring the regional analysis of fractional order DPS with DRONEMATH in mind (e.g. pest spreading in crop fields under drone remote sensing and cropdusting spraying control).

- Fudong Ge, YangQuan Chen and Chunhai Kou. "*Regional Analysis of Time-Fractional Order Diffusion Processes*" (**Springer London,** to appear Winter 2016 or Spring 2017) [draft book ready, 270 pages]
  - JMAA, FCAA, Automatica, IET CTA, IMA JMCI, IJC, IEEE/CAA JAS etc.

UCME MESA LAB

Outline (What else I can do?)

- **Motivations**
- **Why Regional?**
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cyber

- Communication topology
- Communication induced uncertainties

physical

- Space fractional diffusion systems
- Space-time fractional diffusion systems
- Variable order and distributed order fractional diffusion systems
- Nonlinear fractional diffusion systems

systems

- How many actuators/sensors are sufficient when there exists communication among them and how to best configure them for the fractional diffusion control processes?
- The optimal positioning problem of the actuators/sensors;
- Given the desirable zone shape, is it possible to control or contain the fractional diffusion process with static/mobile actuators/sensors within the given zone?

reminder
x 4

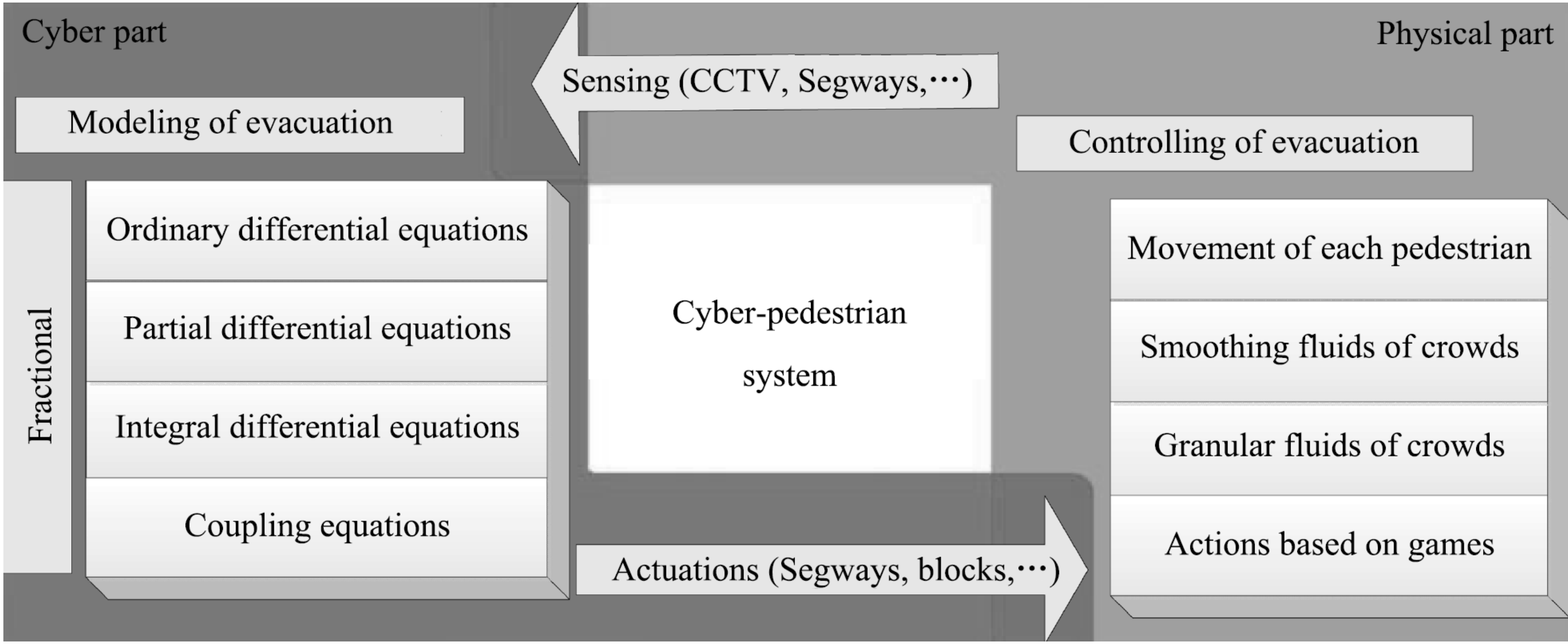
- Process is integer order or fractional order
- Controller is integer order or fractional order

x 4

- Controller is regional or not
- Observer is regional or not

800x16 = 12,800 cases (PhDs)

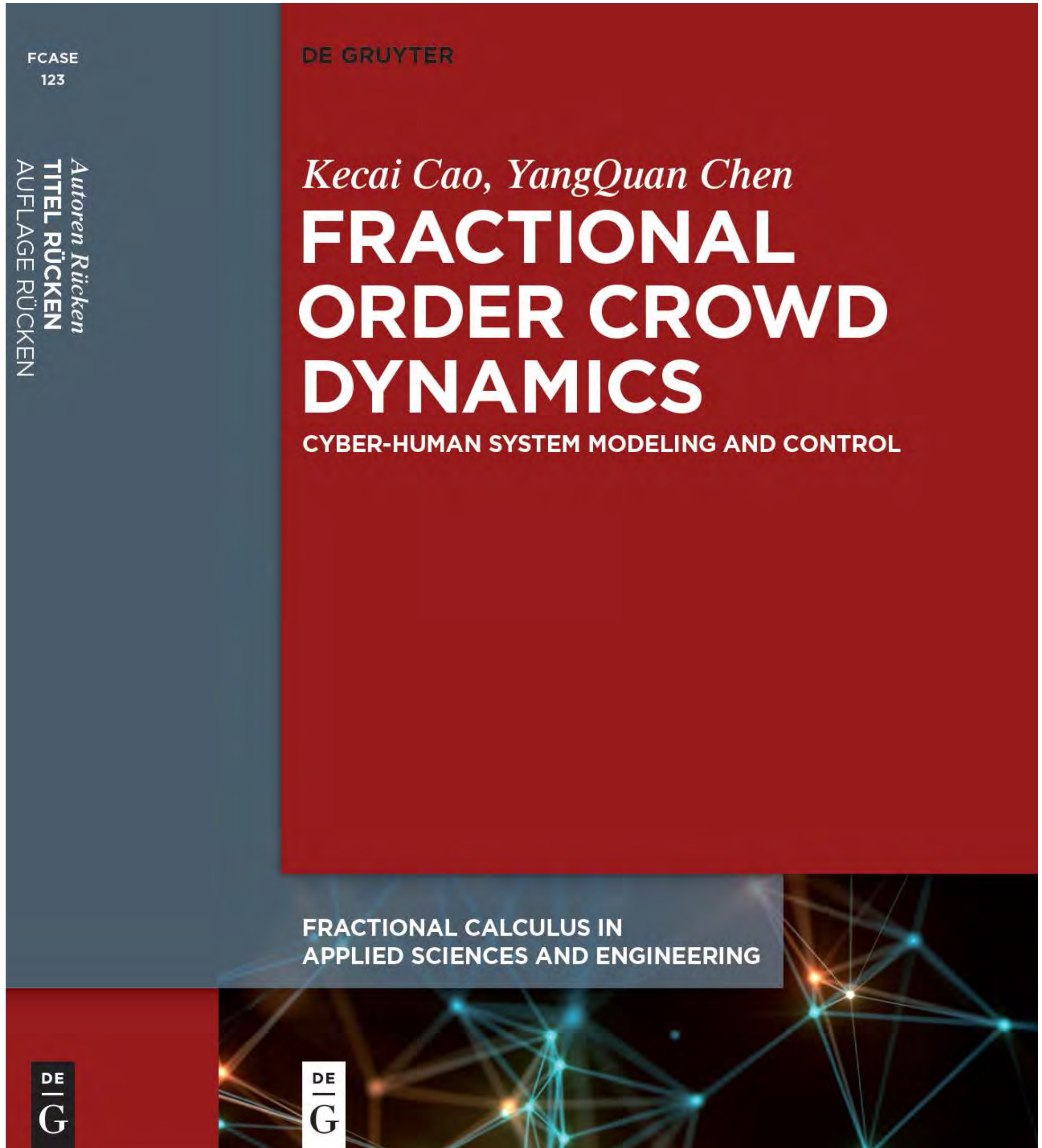
Minds like parachutes function only when open



Kecai Cao, Yangquan Chen, Dan Stuart, and Dong Yue. **Cyber-physical modeling and control of crowd of pedestrians: a review and new framework**. *Automatica Sinica, IEEE/CAA Journal of*, 2(3):334–344, 2015. http://arxiv.org/abs/1506.05340.

New research monographs

- Kecai Cao and YangQuan Chen. "*Fractional Order Crowd Dynamics: Cyber Human System Modeling, and Control*" (Invited book project. Volume #1 of the **De Gryuter Monograph Series "Fractional Calculus in Applied Sciences and Engineering"** ISBN 9783110472813)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Autoren Rücken
TITEL RÜCKEN
AUFLAGE RÜCKEN
DE GRUYTER
Kecai Cao, YangQuan Chen
FRACTIONAL ORDER CROWD DYNAMICS
CYBER-HUMAN SYSTEM MODELING AND CONTROL
FRACTIONAL CALCULUS IN APPLIED SCIENCES AND ENGINEERING
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New robotics research
from controlling crop dynamics to crowd dynamics ...

MAS-net to CPS to CHS: Robots as Sensors and Actuators

Figure: Stampede in Nigeria

Figure: Stampede in Shanghai

In the next a few years ...

UC Multi-campus Synergy on **CIDERS**

California Institute of Data-drone Engineering and Services

UCM, UCSC,UCB, UCSD, LLNL

CIDERS in Scientific data-drones: platforms, operation, and certification

UCM
UCD
UCSD

CIDERS in precision agriculture

UCM
UCD
LBL
SNL
UCSD

CIDERS in environmental monitoring: water, **fire**, soil, dust, AQ …

Thank you for attending my talk!

For more information, check

http://mechatronics.ucmerced.edu/