Statistics Prelim. August 2002

(1) Let $N = 1, 2, ..$ and $\Theta = \{(a,b) : a \leq b, a, b \in N\}$
For each $(a,b) \in \Theta$, Let $P(a) = P(b) = .5$
Let X_1, X_2, X_3 be i.i.d $P_{a,b}$.
Let

$$T(X_1, X_2, X_3) = (X_1, X_2, X_3)/3$$

$$S(X_1, X_2, X_3) = [Max(X_1, X_2, X_3) + Min(X_1, X_2, X_3)]/2$$

Is either of the above a MVUE of $g(a,b) = (a+b)/2$. If not find the MVUE.

Clearly state the results you use and give the details of your argument clearly. [15]

(2) Suppose that $\{f_\theta : \theta \in \theta\}$ is a family of densities such that for all n there exists a MVUE estimate based for θ based on i.i.d observations $X_1, X_2, \ldots, X_n$. If $V_{n,\theta}$ is finite for some n, show that $V_{n,\theta}$ goes to 0 as $n \to \infty$ [8]

(3) Let ϕ be a MP size α test function for testing $H_0 : P$ against $H_1 : Q$. Prove or give a counter example to the following statement:
" $\phi \equiv \alpha$ a.e. $(P + Q)$ iff $P = Q$".[10]

(4) Consider the testing problem $H_0 : \theta \in \{-1, 1\}$ against the alternative $H_1 : \theta = 0$ where θ is the mean of a normal population with variance 1.
  (a) show that there does not exist a UMP test for the above problem based on one observation. [Begin by showing that such a test has to be symmetric][10]
  (b) Does there exist a UMP test based on n observations for some n? Justify your answer.[7]

(5) Let X have the distribution function

$$F_\alpha(x) = 1 - x^{-\alpha}, \quad \alpha > 0$$

  (a) Show that for $\alpha > 2$,

$$E(X) = \frac{\alpha}{(\alpha - 1)} \text{ and } V(X) = \frac{\alpha}{((\alpha-2)(\alpha-1)^2)}$$

  [5]
  (b) Let $Y = (\alpha - 1)X - \alpha$. Show that as $\alpha \to \infty$, Y converges in distribution and identify the limit. [15]

(6) Suppose $X_1, X_2, \ldots, X_n$ is a sample from $U(0, \theta)$. The MLE of θ is M_n– the maximum of $X_1, X_2, \ldots, X_n$.
  (a) Show that $n(\theta - M_n)$ converges in distribution to an exponential distribution [10]
  (b) In view of the above, as an estimate of θ, M_n might not be so good since $M_n < \theta$ with probability 1. Consider the modified estimate $T_n = \frac{(n+c)}{n} M_n$.
    (i) what is the asymptotic distribution of T_n [10]
    (ii) What value of c should be used if we measure accuracy by squared error loss? absolute error loss? [10]

S-2002

(7) Let $X_1, X_2, \ldots,$ be i.i.d random variables with mean μ and variance σ^2. Find the asymptotic distribution of

$$R_n = \frac{\sum_{i=1}^{n} X_{2i-1}}{\sum_{i=1}^{n} X_{2i}}$$

[Deal with the case $\mu = 0$ and $\mu \neq 0$ separately][15]

(8) For $i = 1, 2, \ldots,$, let

$$Y_i = \theta x_i + \epsilon_i$$

where $\epsilon_i, i = 1, 2, \ldots,$ are i.i.d symmetric variables and $x_1, x_2, \ldots$ are non random design values.

Give some sufficient conditions on the model which will ensure the consistency of the least square estimates of θ as n goes to ∞.[15]

(9) Let $X_1, X_2, \cdots, X_n$ be i.i.d observations from $U(0, \theta)$ where $\theta \in \Theta = \{1, 2, 3, \ldots\}$.

(a) find the MLE of θ [10]

(b) Investigate the admissibility of the MLE for 0-1 loss [10]

(c) what can you say about $\sup_\theta R(\theta, T)$ where T is the MLE. [5]

S-2002