STT 871-872 Fall Preliminary Examination
Wednesday, August 26, 2009
12:30 - 5:30 pm

1. Let $X_1, ..., X_n, n > 2$, be iid observations from the exponential distribution $E\left(0, e^\theta\right), \theta \in \mathbb{R}$. Show that the MLE of θ is $\hat{\theta} = \log \overline{X}$ and explicitly find the $\sigma_\theta^2 > 0$ such that $\sqrt{n}(\hat{\theta} - \theta) \rightarrow N(0, \sigma_\theta^2)$, as $n \rightarrow \infty$. **(12)**

2. Let Θ denote the set of integers $\{2, 3, ...\}$. Let $X_1, ..., X_n$ be iid observations from

$$f_\theta(x) = \theta\,(1-x)^{\theta-1}\, I(0 \leq x \leq 1), \quad \text{for some } \theta \in \Theta.$$

(a) Find the MLE $\hat{\theta}$ of the true parameter θ. **(8)**

(b) Show that $\hat{\theta}$ is consistent for θ. **(4)**

3. Let $\Theta := \{(\theta_1, \theta_2, \mu);\ \theta_1 > 0, \theta_2 > 0, \mu \in \mathbb{R}\}$. Let

$$f_{\theta_1,\theta_2,\mu}(x) = \begin{cases} (\theta_1 + \theta_2)^{-1} e^{-(x-\mu)/\theta_1} & x \geq \mu \\ (\theta_1 + \theta_2)^{-1} e^{(x-\mu)/\theta_2} & x < \mu \end{cases}, \quad (\theta_1, \theta_2, \mu) \in \Theta.$$

(a) Show that the family of Lebesgue probability densities $\{f_{\theta_1,\theta_2,\mu}(x) : (\theta_1, \theta_2, \mu) \in \Theta\}$ is complete. **(6)**

(b) Show that UMVUE for μ based on a single observation X from this density does not exist in general. Find a subset of Θ by imposing condition on θ_1, θ_2 such that UMVUE for μ based on X exists. **(6)**

4. Let X be a single observation from $N(\mu, 1)$ where $\mu \in \mathbb{R}$ has the improper Lebesgue prior density $\pi(\mu) = e^\mu$. Under the squared error loss, show that the generalized Bayes estimator of μ is $X + 1$, and that it is neither minimax nor admissible. **(12)**