Chapter 26 Chi-Square Tests

- Goodness of fit: df = #cells of table - 1 (C-1 for cells arranged in a row).
- Homogeneity-Independence: df = (R-1)(C-1). Analyzed the same.
- Homogeneity is when "row counts are sampled separately."
- Chi-Square Statistic is always calculated $\sum_{\text{cells of a table of counts}} \frac{(\text{obs} - \text{exp})^2}{\text{exp}} \geq 0$.
- Significance level (P-value) = probability of getting chi-square statistic that is *at least as large as your data gave* **if the null hpothesis is correct**.
- Using a chi-square table:

	P-value	
df	0.0145	
30	49.34	Prob(chi-sq with df 30 > 49.34) = 0.0145

- Require **all** expected counts ≥ 5. **Not required of observed counts!**
- Can *merge cells* to achieve ≥ 5 requirement.
- Can add independent chi-square statistics to combine experimental results. Add df to get the applicable df for the combined data.
- Remember: If you choose to "reject H_0 whenever P < 0.001" then your type I error probability is 0.001. That is, if H_0 is true then you will "reject H_0" with probability 0.001 (error of type I).
- Chance of error of type II → 0 with lots of data. That is, if H_0 is false you are nearly certain to reject H_0 with enough data.

Goodness of fit example: Is the coin fair?
Suppose we find 63 heads in 100 tosses?

FIRST, WE EMPLOY A Z-TEST FROM CH. 19.

$\mathrm{H}_0 : p = 0.5 \qquad \mathrm{H}_1 : p \neq 0.5$

Data : 63 heads in 100 tosses.

$$\hat{p} = \frac{63}{100} = 0.63, \qquad \hat{q} = 1 - \hat{p} = \frac{37}{100} = 0.37$$

test statistic $= \dfrac{\hat{p} - p_0}{\sqrt{\dfrac{p_0 q_0}{n}}}$ **~ Z if** H_0 **is true (*i.e.* $p = 0.5$)**

q_0 TYPO

Reject if test statistic is too far from 0 (**2 – sided test**).

Test statistic evaluates to $\dfrac{\hat{p} - p_0}{\sqrt{\dfrac{p_0 q_0}{n}}} = \dfrac{0.63 - 0.5}{\sqrt{\dfrac{0.5 \times 0.5}{100}}} = \mathbf{2.60.}$

2-SIDED ALTERNATIVE HYPOTHESIS

$\boldsymbol{P}$ **– value = 2 $\boldsymbol{P}$ (Z > 2.60) = 2 (1 – 0.9953) = 0.0094**. $\approx .01$ (1%)

Conclusion : It is around 1 % likely that *a* fair coin would produce either $\leq$ 37 or $\geq$ 63 heads. **The data does exhibit *a* rarely seen departure from 0.5.**

UNDERSTAND THIS WELL

Is the coin fair? Apply chi-square instead of z-test.

$\}$ χ^2 FROM CH 26

Same data as above (**P = 0.0094**) but analyzed by chi-square.

	heads	tails
expected counts under H_0:	50	50
observed counts	63	37

VERY GOOD AGREEMENT BETWEEN THE CH 19 TEST + CH 26 TEST.

$$\text{chi-square statistic} = \sum_{\text{cells}} \frac{(\text{obs} - \text{exp})^2}{\text{exp}} = 6.76.$$

$$\mathbf{\frac{(63-50)^2}{50} + \frac{(37-50)^2}{50} = 3.38 + 3.38 = 6.76}$$

DF = C − 1 = 2 − 1 = 1 P = **0.0093**

table of chi-sq:	df	0.0093
	1	6.76

χ^2 STATISTIC

a. The P-value, using the z-test of chapter 19, is 0.0094.

b. This closely agrees with the P-value 0.0093 found using the chi-square test of Chapter 26.

Either the coin is fair and this data is "luck of the draw bad" or the coin is not fair. We may never know which.

Can students act like equal probability selectors?
Apply chi-square *goodness of fit* to their choice-data.

H0: choices 1, 2, 3, 4 are equally likely.

H1: not equally likely

DATA FROM 10:20 CLASS

	1	2	3	4	
expected	$\frac{55}{4}$	$\frac{55}{4}$	$\frac{55}{4}$	$\frac{55}{4}$	total 55
observed	8	27	12	8	total 55

WHEN COUNTING # CELLS COUNT ONLY "OBSERVED" CASES.

$$\text{chi-square statistic} = \sum_{\text{cells}} \frac{(\text{obs} - \text{exp})^2}{\text{exp}} = 17.8$$

$$\frac{(8-55/4)^2}{55/4} + \frac{(27-55/4)^2}{55/4} + \frac{(12-55/4)^2}{55/4} + \frac{(8-55/4)^2}{55/4} = \mathbf{17.8}$$

DF = C - 1 = 4 - 1 = 3   P = 0.0005

table of chi-sq:	df	0.0005
	3	**17.73** $\approx \chi^2$ STATISTIC 17.8

If students choose with equal probability, a chi-square at least as large as our 17.8 would only be seen with probability 0.0005. Which is it? We may never know for sure.

Is full moon statistically related to incidence of crime?

χ^2 for "INDEPENDENCE - HOMOGENEITY"
CH26

	FULL MOON	NOT FULL	
violent	2	2	**4**
property	17	21	**38**
drugs	27	19	**46**
abuse	11	14	**25**
other	9	6	**15**
	66	**62**	**138**

$$\frac{66 \cdot 4}{138}$$

expected

$$\begin{pmatrix} 1.91304 & 1.7971 \\ 18.1739 & 17.0725 \\ 22. & 20.6667 \\ 11.9565 & 11.2319 \\ 7.17391 & 6.73913 \end{pmatrix}$$

$(R-1)(C-1)$

df = (5-1)(2-1) = 4

a. Some expected counts are less than 5.
b. Possible "confounding factors."

e.g. MOON PHASES MIGHT COINCIDE WITH HOLIDAYS OR "GAME NIGHTS," AND THUS WITH CRIMES.

Merge cells to meet the "minimum of 5" requirement.

	FULL MOON	NOT FULL		
violent	2	2	**4**	**merge with abuse**
property	17	21	**38**	
drugs	27	19	**46**	
abuse	11+2 =13	14+2 =16	**25+4**	=29
other	9	6	**15**	
	66	**62**	**138**	

$$\frac{66 \cdot 29}{138}$$

expected

$$\begin{pmatrix} 18.1739 & 17.0725 \\ 22. & 20.6667 \\ 13.8696 & 13.029 \\ 7.17391 & 6.73913 \end{pmatrix}$$

new $(R-1)(C-1)$

df = (4-1)(2-1) = 3

$$\text{chi-square statistic} = \sum_{\text{cells}} \frac{(\text{obs} - \text{exp})^2}{\text{exp}} = 3.528 \text{ (merged)}$$

INDEPENDENCE–

P = 0.317 (no evidence against the hpothesis of homogeneity)

Seems that we don't have to worry over confounding factors.

	Hepatitis C	No Hepatitis C	
tatoo parlor	17	35	**52**
tatoo no parlor	8	53	**61**
no tatoo	22	491	**513**
	47	**579**	**626**

EXPECTED

	Hepatitis C	No Hepatitis C	
tatoo parlor	52 47 / 626	52 579 / 626	**52**
	(3.904)	**(48.096)**	
tatoo no parlor	61 47 / 626	61 579 / 626	**61**
	(4.580)	**(56.420)**	
no tatoo	61 513 / 626	513 579 / 626	**513**
	(38.516)	**(474.484)**	
	47	**579**	**626**

Not ≥ 5

$$\text{chi-square statistic} = \sum_{\text{cells}} \frac{(\text{obs} - \text{exp})^2}{\text{exp}} = 57.91$$

$P << 0.0001$

table of chi - sq :	df	0.0001
	(3-1)(2-1) = 2	18.42

But wait! Are all of the expected counts at least 5? No-

Independence/Homogeneity

OBSERVED

MERGE

	Hepatitis C		No Hepatitis C			
tatoo parlor	17	25	35	88	**52**	113
tatoo no parlor	8		53		**61**	
no tatoo	22		491		**513**	
	47		**579**		**626**	

EXPECTED

	Hepatitis C	No Hepatitis C
~~tatoo parlor~~ TATOO ~~tatoo no parlor~~	$\frac{47 \cdot 113}{626} = 8.484$	$\frac{579 \cdot 113}{626} = 104.516$
no tatoo	$\frac{47 \cdot 513}{626} = 38.516$	$\frac{579 \cdot 513}{626} = 474.484$

CHECK THAT MARGINAL TOTALS OF "EXPECTED" TABLE ARE THE SAME AS THE ONES FOR "OBSERVED" TABLE.

chi – square statistic e.g. $\frac{47 \cdot 113}{626} + \frac{47 \cdot 513}{626} = \frac{47 \cdot 626}{626} = 47$

$\chi^2 = \sum \frac{(obs - exp)^2}{exp} = 42.39$

P

table of chi - sq : df .0001

$(R-1)(C-1) = (2-1)(2-1) = 1$ 15.14 LARGEST ENTRY OFF THE TABLE!

Are all of the expected counts at least 5? YES. $P < .0001$. RARE.