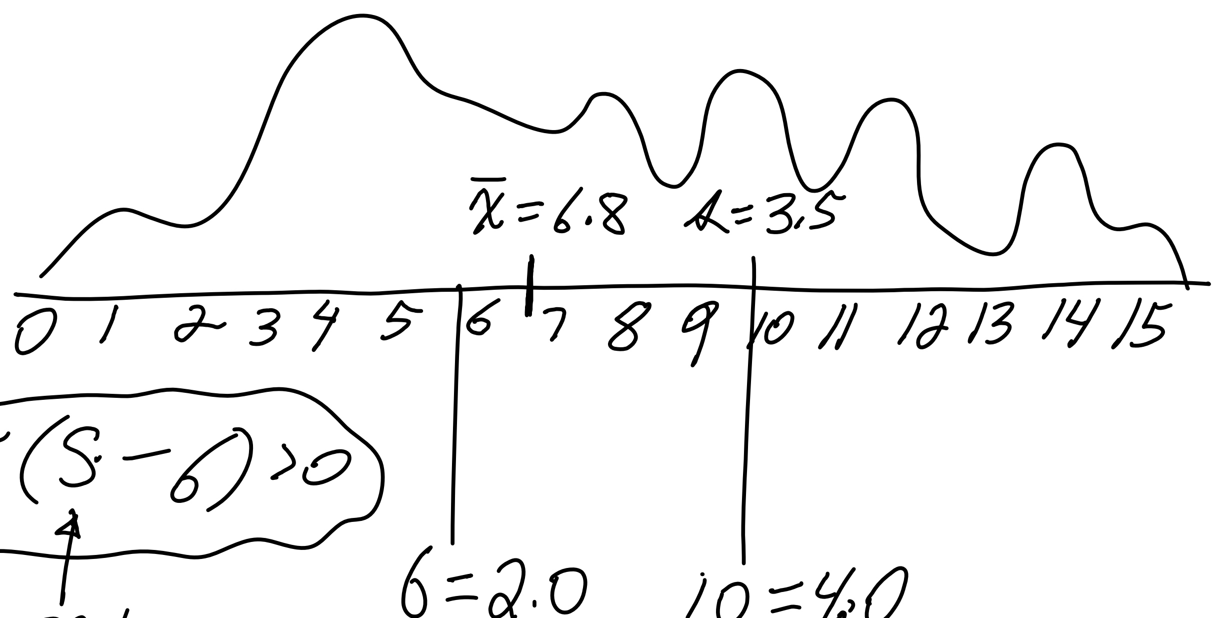
STT200 4-3-09a

NOTE: ON ANGEL THERE IS A COLUMN SHOWING "0 OR 2" – ATTENDANCE 3-16, 18, 20.

EXAM 3



$$\boxed{G = 2 + 0.5(S - 6) > 0}$$

ATTENDANCE CAN BUMP +2 $\rightarrow$ RAW SCORE

WHY LEARN ABOUT GAME RET X

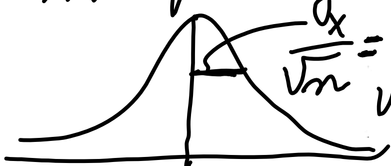
$EX = 6.2$

$\sigma_x = 8.4$

PLAY 100 TIMES

$X_1 \ X_2 \cdots X_{100}$

INDEP RETURNS



$\frac{\sigma_x}{\sqrt{n}} = \frac{8.4}{\sqrt{100}}$

$EX = 6.2$ $\bar{X}$

AVG RET FOR 100 PLAYS

WRITE: $E(\bar{X}) = EX_1 = 6.2$

ALSO $\sigma_{\bar{x}} = \sigma_x / \sqrt{n} = \frac{8.4}{10} = .84$

A Billion Dollar Misunderstanding? In the late 1990s the Bill and Melinda Gates Foundation began funding an effort to encourage the breakup of large schools into smaller schools. Why? It had been noticed that smaller schools were more common among the best-performing schools than one would expect. In time, the Annenberg Foundation, the Carnegie Corporation, the Center for Collaborative Education, the Center for School Change, Harvard's Change Leadership Group, the Open Society Institute, Pew Charitable Trusts, and the U.S. Department of Education's Smaller Learning Communities Program all supported the effort. Well over a billion dollars was spent to make schools smaller.

But was it all based on a misunderstanding of sampling distributions? Statisticians Howard Wainer and Harris Zwerling[13] looked at the mean test scores of schools in Pennsylvania. They found that indeed 12% of the top-scoring 50 schools were from the smallest 3% of Pennsylvania schools—substantially more than the 3% we'd naively expect. But then they looked at the *bottom* 50. There they found that 18% were small schools! The explanation? Mean test scores are, well, means. We are looking at a rough real-world simulation in which each school is a trial. Even if all Pennsylvania schools were equivalent, we'd expect their mean scores to vary. How much? The CLT tells us that means of test scores vary according to $\frac{\sigma}{\sqrt{n}}$. Smaller schools have (by definition) smaller n's, so the sampling distributions of their mean scores naturally have larger standard deviations. It's natural, then, that small schools have both higher and lower mean scores.

On October 26, 2005, *The Seattle Times* reported:

> *[T]he Gates Foundation announced last week it is moving away from its emphasis on converting large high schools into smaller ones and instead giving grants to specially selected school districts with a track record of academic improvement and effective leadership. Education leaders at the Foundation said they concluded that improving classroom instruction and mobilizing the resources of an entire district were more important first steps to improving high schools than breaking down the size.*

ISSUE: SAY HAVE POP^N OF H.S. STUDENTS

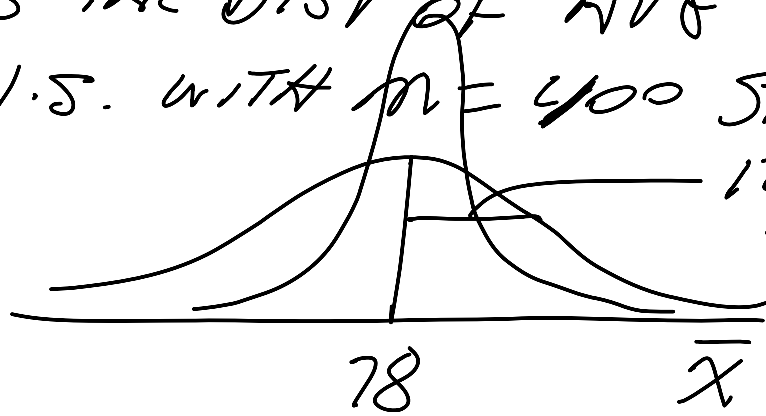
RANDOMLY SELECT 400 FOR A SCHOOL.

LET OVERALL POP^N AVG SCORE BE $\mu_{pop} = 78$

& WITH POP^N S.D. $\sigma_{pop} = 12$

WHAT IS THE DIST OF AVG TEST SCORE IN THIS H.S. WITH $n = 400$ STUDENTS.

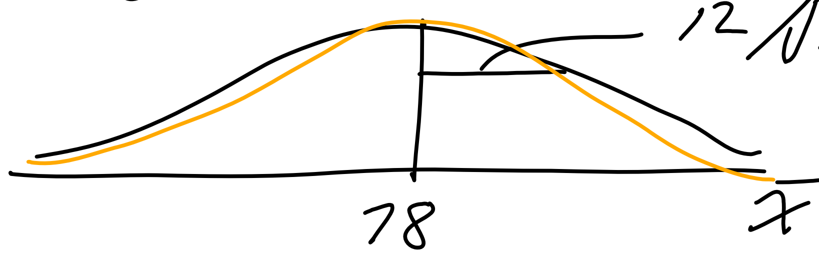
$\sim$



$12/\sqrt{400} = 12/20 = .6$

AVG SCORE OF THAT H.S.'s 400 STUDENTS.

DO H.S. WITH $n = 2500$



$12/\sqrt{2500} = \frac{12}{50} = .24$

$n = 2500$

WALLACE & ROBERTS.
(PUBL. FREE PRESS)

DO THE MATH:

RULES: $E(X+Y) = EX + EY$ } ALWAYS
$Ec = c \quad EcX = cEX$

IF X, Y ARE INDEP (MEANS THAT IF YOU ARE TOLD eg $X = 7.2$ THE DIST^N OF Y NEED NOT BE CHANGED TO REFLECT THAT INFORMATION).

THEN $E(XY) = (EX)(EY)$

AND $\mathrm{Var}(X+Y) = \mathrm{Var}\,X + \mathrm{Var}\,Y$

Proof: Spse $EX = EY = 0$

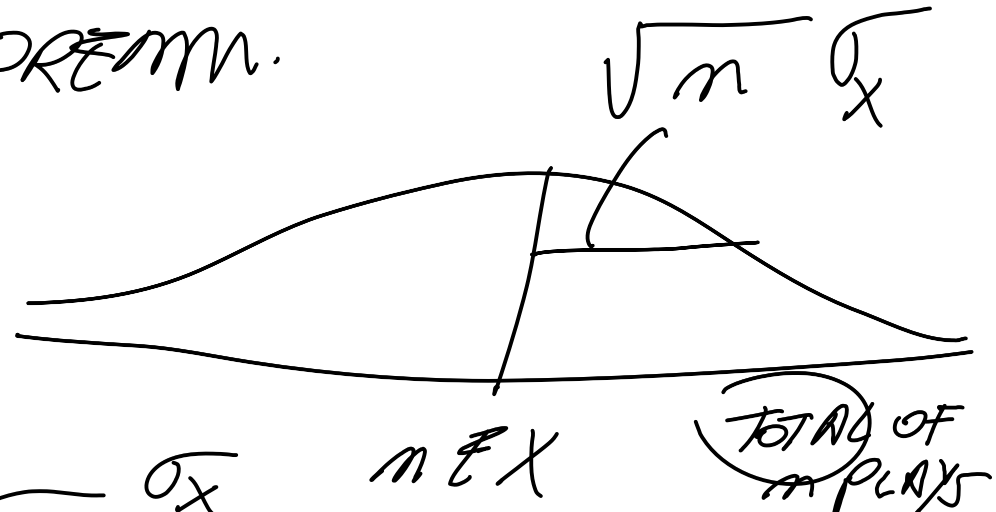
$$\mathrm{Var}(X+Y) \underset{\text{CASE}}{\overset{\text{THIS}}{=}} E(X+Y)^2$$
$$= E(X^2 + 2XY + Y^2)$$
$$= E(X^2) + \underbrace{2E(XY)}_{} + E(Y^2)$$
$$\underset{\text{CASE}}{=} \mathrm{Var}\,X \quad \underbrace{2E(X)(EY)}_{0} \quad \mathrm{Var}\,Y$$
$$= \mathrm{Var}\,X + \mathrm{Var}\,Y$$

So $\mathrm{Var}(\bar{X}) \overset{\text{INDEP } X_1 + \cdot + X_n}{=} \mathrm{Var}\left(\frac{X_1 + \cdot + X_n}{n}\right)$

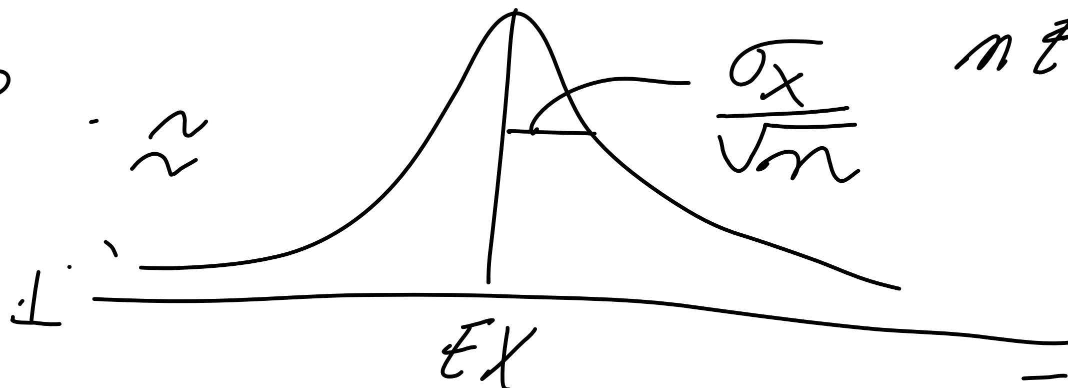
$$= \frac{}{n^2}\mathrm{Var}(X_1 + \cdot + X_n)$$
$$= \frac{1}{n^2}(\mathrm{Var}\,X_1 + \mathrm{Var}\,X_2 + \cdot + \mathrm{Var}\,X_n)$$
$$= \frac{n}{n^2}\mathrm{Var}\,X = \frac{\mathrm{Var}\,X}{n}$$

CENTRAL LIMIT THEOREM.

TRYING TO SAY THAT $\sim$



ALSO $\sim$



$\bar{x} = \frac{\text{TOTAL}}{n}$

TECH STATEMENT

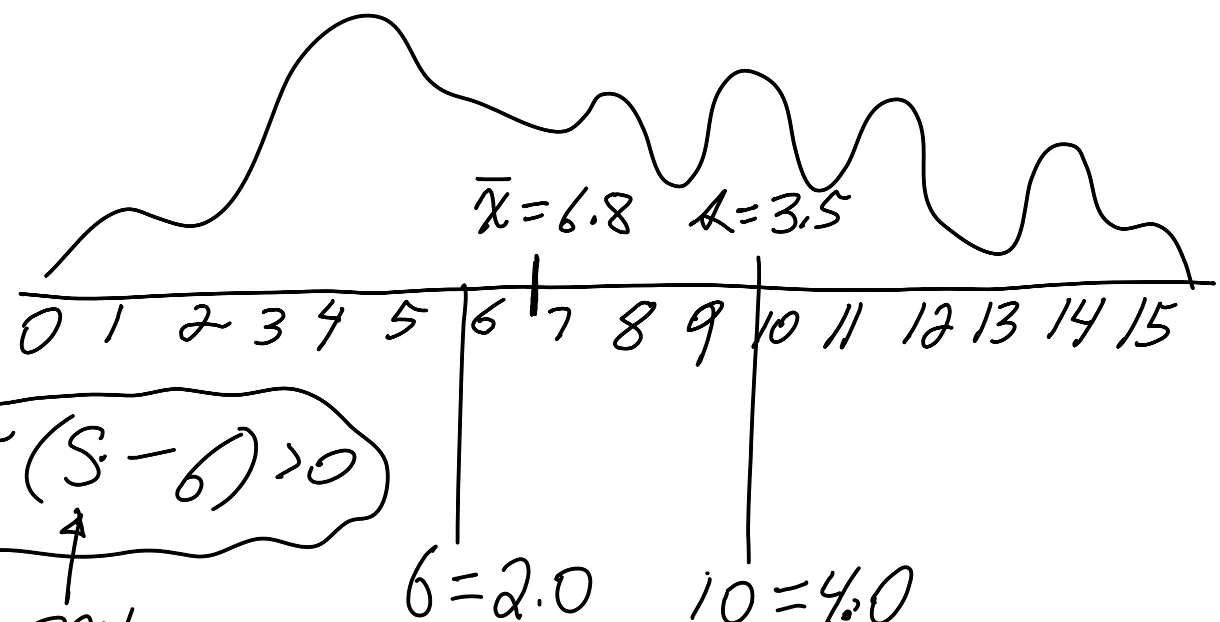
$$P\left(\frac{\bar{X} - EX}{\sigma_X / \sqrt{n}} \le z \right) \xrightarrow[n \to \infty]{} $$

any z

$$= \int_{-\infty}^{z} dz \, \frac{1}{\sqrt{2\pi}} e^{-z^2/2}$$

LIKE STD NORMAL Z

EXAM 3



$$G = 2 + 0.5(S - 6) > 0$$

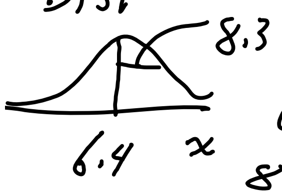
RAW SCORE

IMAGINE LOTTERY X

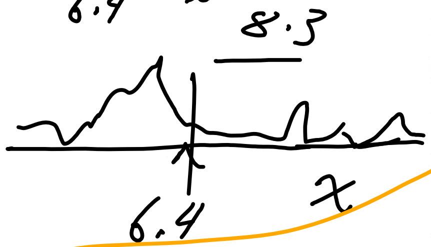
$EX = 6.4, \ \sigma_X = 8.3$

COULD BE DIST

ONE PLAY

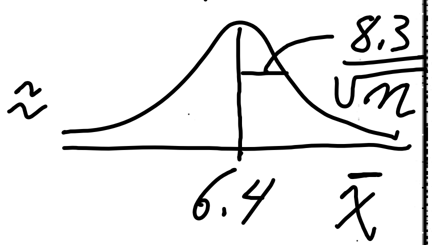


or



AVG $\bar{X}$ OF INDEP PLAYS $X_1 \dots X_n$

$\approx$



A Billion Dollar Misunderstanding? In the late 1990s the Bill and Melinda Gates Foundation began funding an effort to encourage the breakup of large schools into smaller schools. Why? It had been noticed that smaller schools were more common among the best-performing schools than one would expect. In time, the Annenberg Foundation, the Carnegie Corporation, the Center for Collaborative Education, the Center for School Change, Harvard's Change Leadership Group, the Open Society Institute, Pew Charitable Trusts, and the U.S. Department of Education's Smaller Learning Communities Program all supported the effort. Well over a billion dollars was spent to make schools smaller.

But was it all based on a misunderstanding of sampling distributions? Statisticians Howard Wainer and Harris Zwerling[13] looked at the mean test scores of schools in Pennsylvania. They found that indeed 12% of the top-scoring 50 schools were from the smallest 3% of Pennsylvania schools—substantially more than the 3% we'd naively expect. But then they looked at the *bottom* 50. There they found that 18% were small schools! The explanation? Mean test scores are, well, means. We are looking at a rough real-world simulation in which each school is a trial. Even if all Pennsylvania schools were equivalent, we'd expect their mean scores to vary. How much? The CLT tells us that means of test scores vary according to $\frac{\sigma}{\sqrt{n}}$. Smaller schools have (by definition) smaller n's, so the sampling distributions of their mean scores naturally have larger standard deviations. It's natural, then, that small schools have both higher and lower mean scores.

On October 26, 2005, *The Seattle Times* reported:

> *[T]he Gates Foundation announced last week it is moving away from its emphasis on converting large high schools into smaller ones and instead giving grants to specially selected school districts with a track record of academic improvement and effective leadership. Education leaders at the Foundation said they concluded that improving classroom instruction and mobilizing the resources of an entire district were more important first steps to improving high schools than breaking down the size.*

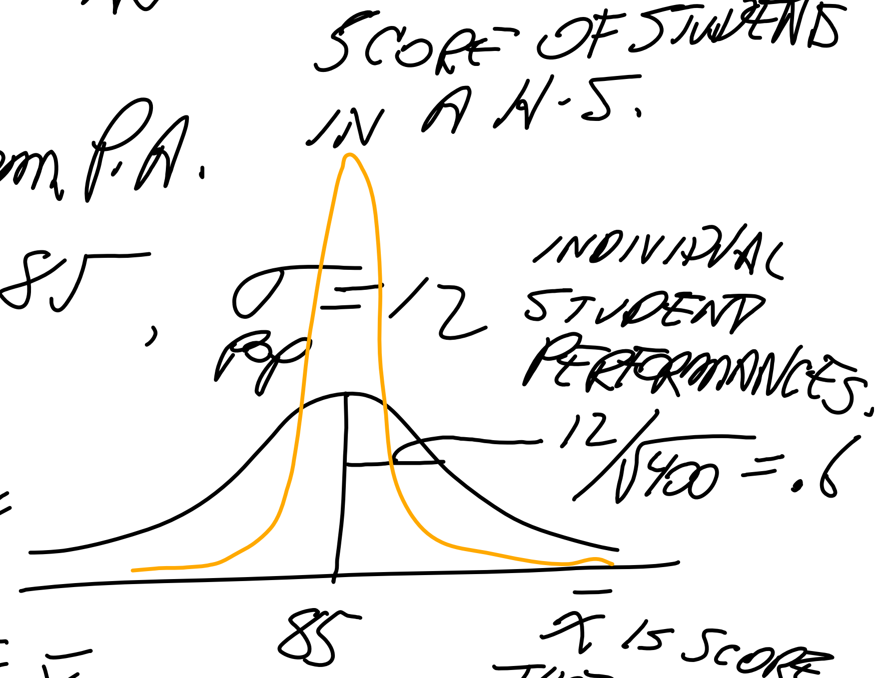
TO BE CLEAR: $\bar{X} = \frac{X_1 + \cdots + X_n}{n}$ (RANDOM) AVG

SUPPOSE H.S. STUDENTS ARE SELECTED AT RANDOM FROM P.A.

SUPPOSE $\mu_{Pop\,PA}$ = POP AVG = 85, $\sigma_{Pop} = 12$

"MU"

CASE OF HS $n = 400 \Rightarrow \approx$



THIS CURVE REPRESENTS THE PROSPECTS FOR THIS HS'S $\bar{X}$

ALSO - THE LIKELY VARIATION AMONG $\bar{X}$'S FOR ALL HS OF 400 STUDENTS.

VS HS W/ $n = 2500$ $(= 50^2)$

RECALL RULES: $\underbrace{E(X+Y) = EX + EY, \quad Ec = c, \quad EcX = cEX}_{\text{ALWAYS}}$.

IF X, Y ARE **INDEPENDENT** (SAYS KNOWING, SAY, THAT $X = 6.4$ TELLS US NOTHING ABOUT Y)

THEN $\operatorname{Var}(X+Y) = \operatorname{Var} X + \operatorname{Var} Y$

(END CH 16)

GEN'L $\operatorname{Var}(X+Y) = \operatorname{Var} X + 2\operatorname{Cov}(X,Y) + \operatorname{Var} Y$

$$= \operatorname{Var} X + \underbrace{2\operatorname{Corr}(X,Y)\,\sigma_X\,\sigma_Y} + \operatorname{Var} Y$$

If X IND Y $\Rightarrow \operatorname{Corr}(X,Y) = 0$ SO MIDDLE TERM VANISHES & GET RULE AT TOP OF PAGE.

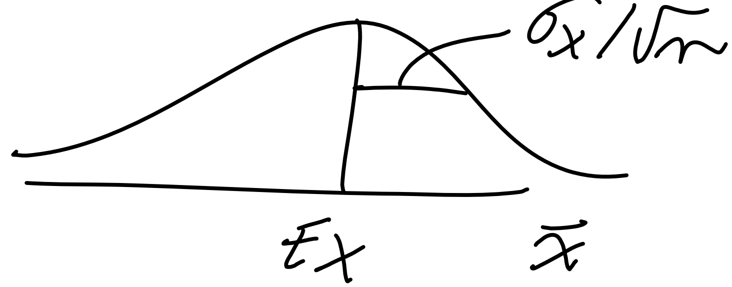
WHY? SUPPOSE $EX = EY = 0$

$$Var(X+Y) \overset{\text{THIS}}{\underset{\text{CASE}}{=}} E(X+Y)^2 = E(X^2 + 2XY + Y^2)$$

$$= Var X + \underbrace{2E(XY)}_{2Corr(X,Y)\sigma_X\sigma_Y} + Var Y$$

STATEMENT OF CENTRAL LIMIT THEOREM (ONLY)

SAY DIST OF $\bar{X} \approx$



$$P\left(\frac{\bar{X} - EX_1}{\sigma_x/\sqrt{n}} < z\right) \xrightarrow[n\to\infty]{}$$

$\approx Z$ STD NORMAL