STT 200 7-16-10 8am.

Note Title 7/15/2010

TODAY, USE NORMAL DISTRIBUTIONS
AS THEY ARE — eg IN NATURAL WORLD MEASUREMENTS, BUSINESS ACTIVITIES & SCIENCE (OFTEN FORCED BY QUALITY ASSURANCE METHODOLOGY), APPROX OF OTHER DISTRIBUTIONS.

FIRST NORMAL DISTRIBUTIONS, z-SCORES, TABLE pg 210.

LOOK AT TABLE.

$\underbrace{\quad}_{68\%}$ z SCORE 0 ± 1

$z = 1.00$

z	.00	.02
1.0	.3413	

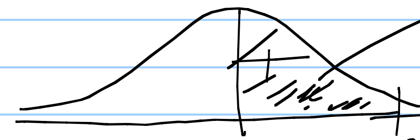
NOTE: eg IQ $\mu = 100$
$\sigma = 1$

$\underbrace{\quad}_{68\%}$ $100 \pm \sigma$

$$\text{SO } P(\text{IQ IN RANGE } \underset{\mu}{100} \text{ TO } \underset{\mu+\sigma}{115}) \approx .3413$$

LIKEWISE

z	$\underline{\underline{.06}}$
1.9	0.4750



$\frac{.95}{2} = .475$

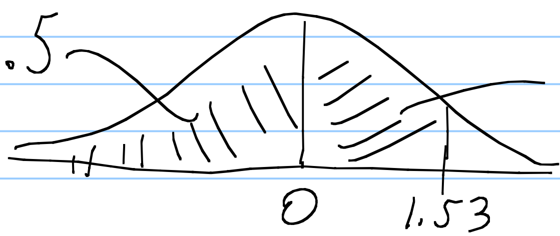
$1.96 = z$

95%

ANOTHER USE OF TABLE.

$$\text{FIND } P(\text{IQ} < 123) \underset{\text{DIST}}{\overset{\text{NORMAL}}{=}} P\left(Z < \frac{123 - \overset{\mu_{IQ}}{100}}{\underset{\sigma_{IQ}}{15}}\right)$$

$$= P\left(Z < \frac{23}{15}\right) = P(Z < \underline{\underline{1.53}})$$



.5

TABLE GIVES z TAILS

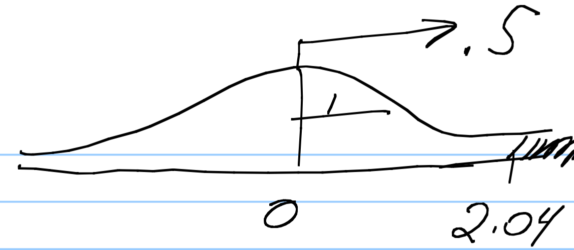
z	$\underline{\underline{.03}}$
1.5	$\boxed{0.437}$

$\underline{\underline{\text{ANS}}}$ $.5 + 0.437$

$= .937$

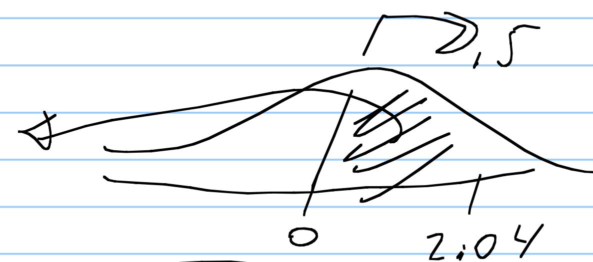
YET ANOTHER EXAMPLE OF TABLE USE.

$P(Z > 2.04) = .5 - .4793 = .0207$



.5 – BETWEEN 0 AND 2.04

z	.04
2.0	$\boxed{0.4793}$

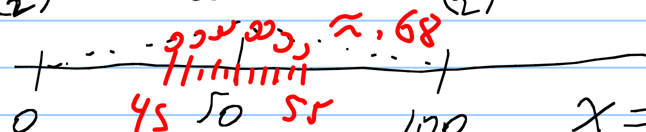


BINOMIAL IS APPROXIMATED BY NORMAL $\mu = np$

eg $n = 100$, $p = \frac{1}{2}$ (100 COIN TOSSES) $\sigma = \sqrt{np(1-p)}$

$\mu = 100\left(\frac{1}{2}\right) = 50$ $\sigma = \sqrt{100\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)} = 5$.

TRUE DIST^N $\left(\frac{1}{2}\right)^{100}$ $\frac{100!}{50!50!}\left(\frac{1}{2}\right)^{100}$ $\left(\frac{1}{2}\right)^{100}$

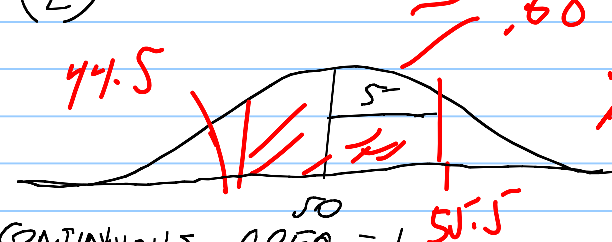


$x = \#H$

$\approx$

DISCRETE Sum OF PROBABILITIES = 1

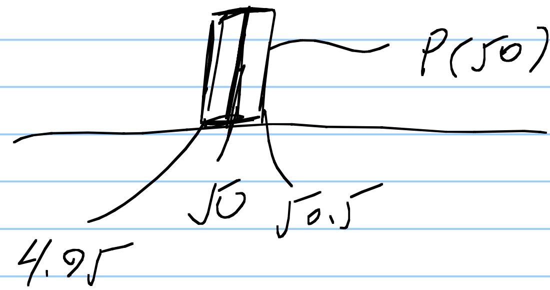
$\approx .68$ LATER DO MORE ACCURATELY



CONTINUOUS AREA = 1

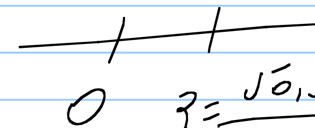
ANOTHER

P(GET EXACTLY 50 H) ~ P(CONTINUOUS BETWEEN 49.5 + 50.5)



$$\frac{100!}{50!\,50!}\left(\frac{1}{2}\right)^{50}$$

OR TWICE



$$z = \frac{50.5 - 50}{5} = .1$$

z	0.00
.1	$\boxed{.0398}$

ANS $P(50) = 2(.0398) = .0796$

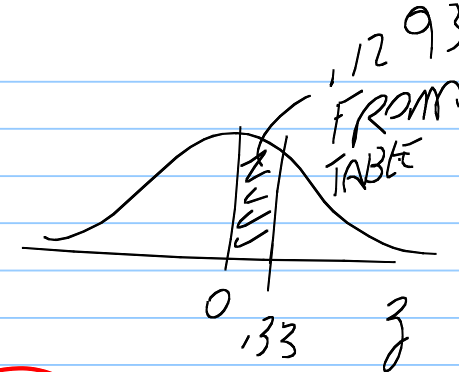
.0796

COOL! "TOP LEVEL THINKING"

PAUSE TO LOOK AT #1 FROM HW 7-19-10

GIVE NORMAL $\mu = 1.77$ $\sigma = .09$

(e) STD SCORE OF 1.8 IS $z = \dfrac{1.8 - 1.77}{.09} = .33$



POISSON DISTRIBUTION FOR COUNTS OF RARE EVENTS.

AND ITS APPROXIMATION BY NORMAL.

THINK FIRST OF BINOMIAL $p(x) = \dfrac{n!}{x!(n-x)!} p^x (1-p)^{n-x}$

n LARGE
p SMALL $\Rightarrow p(x) \approx$ DEPENDS ONLY ON np

eg TRY 500 TIMES ($n = 500$) WITH $p = \frac{1}{100}$ EXPECT $np = 5$.
ON AVE.

$$p(3) = \frac{500!}{3!\,497!}\left(\frac{1}{100}\right)^3\left(\frac{99}{100}\right)^{497}$$

$x = 3$

INSTEAD TRY $n = 5000$, $p = \frac{1}{1000}$ AGAIN $np = 5000\left(\frac{1}{1000}\right) = 5$

$$p(3) = \frac{5000!}{3!\,4997!}\left(\frac{1}{1000}\right)^3\left(\frac{999}{1000}\right)^{4997}$$

ALMOST THE SAME!!

$$\approx e^{-np}(np)^x / x!$$

$$= e^{-5}\,\frac{5^3}{3!} = e^{-5}\left(\frac{125}{6}\right)$$

$e = 2.718281828$

EXAMPLE. WE AVG 2.3 EMPTY SEATS PER FLIGHT (EXPERIENCE)

IF POISSON IS APPLICABLE

P(GIVEN FLIGHT HAS 3 EMPTY SEATS)

$$\sim e^{-2.3} (2.3)^3 / 3!$$

EXAMPLE. COOKIES.

144 BAKER'S DOZEN

6×144 RAISINS. = 864

$p = \frac{1}{144}$ $\quad n = 864$

$np =$ (WANTED) 6

$3! = 3 \cdot 2 \cdot 1 = 6$

$P(\text{GIVEN COOKIE HAS 3 RAISINS}) \underset{\text{POISSON}}{\approx} e^{-6}\dfrac{6^3}{3!} = .089$

$6^0 = 1$

$0! = 1$

$P(\text{GIVEN COOKIE HAS 0 RAISINS}) = e^{-6}\dfrac{6^0}{0!}$

<u>EXAMPLE</u> WE AVG 4.8 LIGHTNING STRIKES PER SEASON

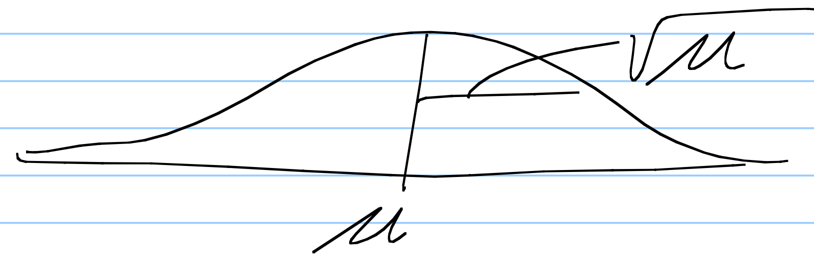
$P(\text{GET NONE THIS SEASON}) \underset{\text{IF POISSON}}{\approx} e^{-4.8}\dfrac{4.8^0}{0!} = e^{-4.8}$

$\approx .0082$

NORMAL APPOX OF POISSON

IF MEAN OF POISSON IS ≥ 10 (SAY) CALL IT μ

POISSON $\approx$ NORMAL



WHY $\sqrt{\mu}$?

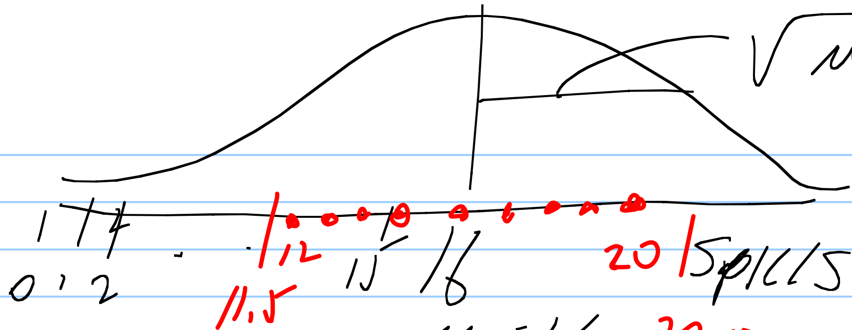
THINK $\sqrt{np(1-p)}$ RARE EVENTS

$\sim \sqrt{np} = \sqrt{\mu}$ $p \sim 0$

EXAMPLE: NORMAL APPROX OF POISSON $\mu \geq 10$ RULE WE INVOKE !!

SUPPOSE WE AVG 16 OIL SPILL INCIDENTS PER YEAR.
THINK POISSON BECAUSE IT DEALS WITH COUNTS OF RARE EVENTS <u>MAY</u> BE APPLICABLE: — SUPPOSITION! CHECK LATER

$\approx$ NORMAL APPROX



$\sqrt{\mu} = 4$

$\mu = 16$

20.5 MORE ACCURATE

$\approx$ 68% CHANCE
CRUDE GET
APPROX 12 TO 20.