

Bonus quiz 7-21-10 8am

Probability, Random Variables, Expectation, Std Deviation

DO NOT REDUCE ANSWERS.
LEAVE SUMS, PRODUCTS, FRACTIONS UNEVALUATED.
I NEED TO WITNESS YOUR REASONING.

1. A box has colored balls: 10 R, 6 G. Successive draws are without replacement and with equal probability on the balls then remaining.

a. $P(G_2)$ using rules of probability (give all important steps) $6/16 = 3/8$

$$P(G_2) = P(R_1 \text{ and } G_2) + P(G_1 \text{ and } G_2) - 0 \text{ (no overlap)}$$

$$= P(R_1) P(G_2 | R_1) + P(G_1) P(G_2 | G_1)$$

$$= \frac{10}{16} \cdot \frac{6}{15} + \frac{6}{16} \cdot \frac{5}{15} = \frac{90}{240} = \frac{9}{24} = \frac{3}{8}$$

b. If INSTEAD we **replace** ball 1 before the second draw (give all important steps)

$$P(G_2) = P(R_1 \text{ and } G_2) + P(G_1 \text{ and } G_2) - 0 \text{ (no overlap)}$$

$$= P(R_1) P(G_2 | R_1) + P(G_1) P(G_2 | G_1)$$

$$= \frac{10}{16} \cdot \frac{6}{16} + \frac{6}{16} \cdot \frac{5}{16} = \frac{90}{256}$$

2. A red cube is marked with {2, 2, 5, 5, 9, 15}. A green cube is marked with {4, 6, 6, 7, 7, 13}.

a. $P(R < G)$.

$$23/36$$

b. $P(R < G | R=2) = \frac{4}{6} = 100\%$

R	4	6	6	7	7	13
2	2	2	2	2	2	2
2	2	2	2	2	2	2
5	5	5	5	5	5	5
5	5	5	5	5	5	5
9	9	9	9	9	9	9
15	15	15	15	15	15	15

c. $P(R + G = 9)$. $4+5=9 = 2/36$

$$6+3=9 = 2/36$$

$$\frac{4}{36} = \frac{1}{9}$$

3. A bent coin turns up heads with probability 0.4. The coin is tossed 12 times.

a. The number of ways that one may arrange 3 letters H and three letters T into a line.

$$C(6, 3) = \frac{6!}{3!3!} = 20 \quad \text{HHH TTT}$$

b. The probability of the sequence HTH.

$$.4(.6)(.4) = .096$$

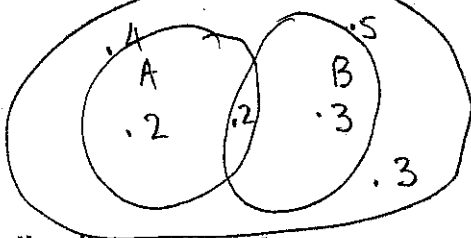
c. The aggregate probability of all sequences that have two H and one T.

$$C(3, 2) \cdot (.096) = \frac{3!}{2!1!} \cdot (.096) = .288$$

4. Events A, B have

$$P(A) = 0.4 \quad P(B) = 0.5 \quad P(A \text{ and } B) = 0.2$$

Complete a Venn diagram for the above information.



5. For the distribution of r.v. X:

x	0	4	5
p(x)	0.1	0.2	0.7

a. $EX = 0(.1) + 4(.2) + 5(.7) = 4.3$

b. Standard deviation X.

$$\sigma^2 = EX^2 - (EX)^2$$

$$EX^2 = 0^2(.1) + 4^2(.2) + 5^2(.7) = 20.7$$

$$\sigma^2 = 20.7 - 4.3^2 = 2.21$$

$$\sigma = \sqrt{\sigma^2} = \sqrt{2.21} = 1.487$$

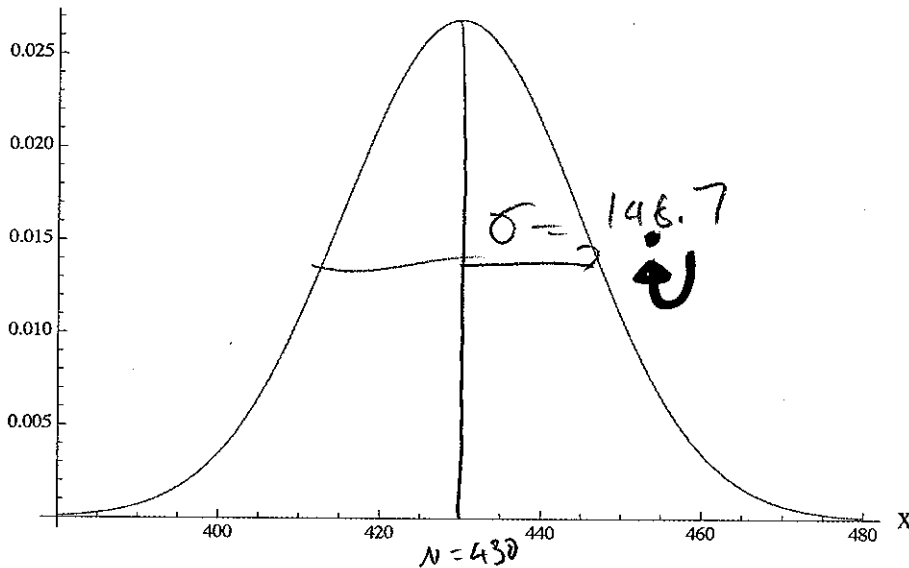
If X_1 and X_2 are **independent** plays from the above distribution for X :

c. $E(3X - 2Y + 5)$ (independence not required)

$$= 3EX - 2EY + 5.$$

d. $\text{Var}(3X - 2Y + 5) = 3^2 \text{Var}(X) + (-2)^2 \text{Var}(Y)$
 $= 9 \text{Var}(X) + 4 \text{Var}(Y)$

e. Sketch the normal approximation of the distribution of the total of 100 independent plays of X . Identify (a) and (b) as recognizable elements of your sketch.



$$100EX = 100(4.3) = 430$$

$$100(1.487) = 148.7$$