STT351-002
EXAM 1 KEY

002

1. a. Given independent random variables X, Y with Var X = 3, Var Y = 2. Give the value of Var(5 X + 2 Y − 7) . Do not reduce.

$$\mathrm{Var}(5X+2Y-\cancel{7}) \overset{\text{INDEP}}{=} \mathrm{Var}(5X) + \mathrm{Var}(2Y)$$
$$= 25\,\mathrm{Var}\,X + 4\,\mathrm{Var}\,Y$$
$$= 25(3) + 4(2)$$

b. Unrelated to (a). For the discrete probability density

x	p(x)	$x\,p(x)$	$x^2 p(x)$
0	1/4	0	0
2	1/4	2/4	4/4 = 1
8	1/2	8/2	64/2 = 32
		$EX = 4.5$	$EX^2 = 33$

$$\mathrm{Var}\,X = EX^2 - (EX)^2 = 33 - 4.5^2$$

Set up and numerically evaluate Variance X. Do not reduce it.

2. Recall that the mean of the binomial distribution is np and the variance is np(1-p).

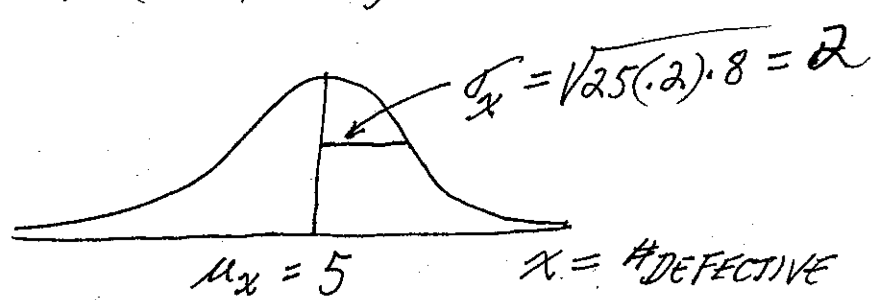
a. Sketch the CLT-approximation of the distribution of random variable X = the number of defective items in a shipment of 25 items. Assume each item is defective with probability 0.2 and items are independent. Be sure to identify the mean and standard deviation as recognizable numerical elements in your sketch. Show calculations.

$\mu_X = EX = np = 25(0.2) = 5$

$\text{Var}\, X = np(1-p) = 25(0.2)(0.8) = 25\ 0.16$

$\sigma_X = 5(0.4) = 2$

$\approx$



b. Refer to (a). Determine the normal approximation of the probability of having **fewer than 8** defective items in the sample of 25 items. It is our custom to instead approximate P(X < 7.5) (i.e. use the continuity correction). Calculate the relevant z-score and use it to obtain the normal approximation of this probability.

$P(X < 7.5) \approx P\left(Z < \frac{7.5-\mu_X}{\sigma_X}\right)$

$= P\left(Z < \frac{7.5-5}{2}\right) = P(Z < 1.25) = 0.8944$

z .05

1.2 $\boxed{0.8944}$

close!

NOTE: $\binom{25}{0}.2^0.8^{25} + \binom{25}{1}.2^1.8^{24} + \cdots + \binom{25}{7}.2^7.8^{18} = 0.8909$

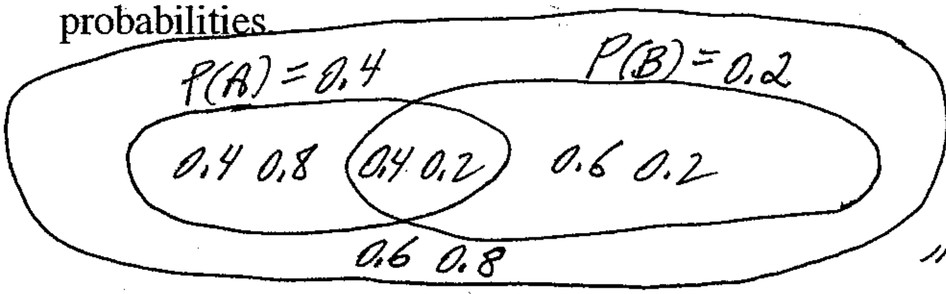
p(0) p(1) p(7)

IS EXACT BINOMIAL PROBABILITY $P(X < 8)$.

3. A, B are independent events with P(A) = 0.4, P(B) = 0.2.

a. Determine P(A | B^C). $P(A|B^C) \overset{INDEP}{=} P(A) = 0.4$

b. Complete a Venn diagram with all four regions and their probabilities.



$P(A^C) = 0.6$

$P(B^C) = 0.8$

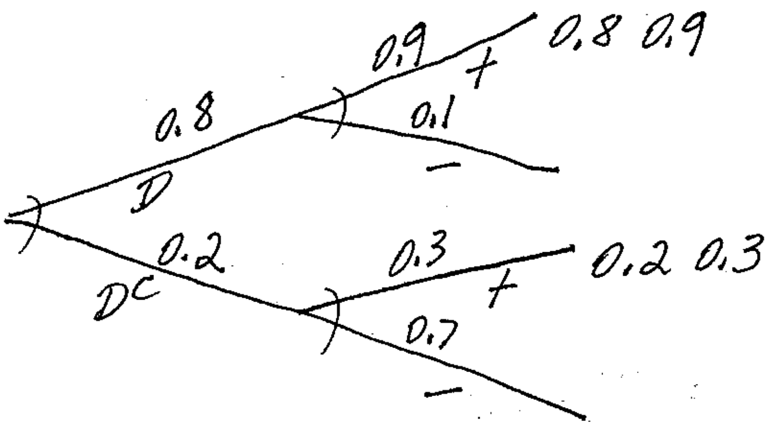
eg $P(AB^C)$

$\overset{INDEP}{=} P(A)P(B^C)$

$= 0.4\ 0.8$

4. TREE. 80% of castings are defective.
90% of defective castings test positive for defect.
30% of non-defective castings test positive for defect.

a. Determine the probability that a casting tests **positive**.



$P(+) = 0.8\ 0.9 + 0.2\ 0.3$

b. Determine P(casting is defective | test is **positive**). Do not reduce.

$$P(D|+) = P(D+)/P(+)$$

$$= \frac{0.8\ 0.9}{0.8\ 0.9 + 0.2\ 0.3}$$

5. Battery life in hours x is modeled as a random variable X with $P(X > x) = 1/x^2$ for $x > 1$.

a. Determine the probability that a battery will live longer than 3 hours, i.e. $P(X > 3)$.

$P(X>3) = 1/3^2 = 1/9$, $x=3$

b. Determine the conditional probability that a battery will live an **additional** 3 hours if working at 5 hours, i.e. $P(X > 8 \mid X > 5)$.

$P(X>8 \overset{\text{and}}{\cap} X>5)/P(X>5) = P(X>8)/P(X>5)$

$= (1/8^2)/(1/5^2) = (5/8)^2$

6. A process produces ball bearings scored by x = hardness. Assume that E X = 3.5 with s.d. = sigma = 0.2 and the process is in statistical control.

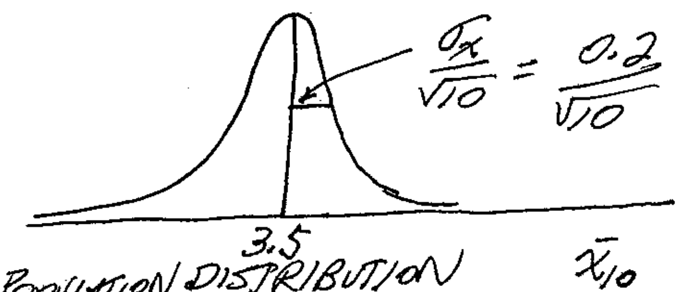
a. Denote by $xBAR_{10}$ the sample average of a with-replacement sample of 10 such parts. Determine numerically, but do not reduce,

E $xBAR_{10}$ $= \mu_{\bar{x}_{10}} = \mu_x = 3.5$

STATISTICAL CONTROL $\Rightarrow$ INDEP

s.d. $xBAR_{10}$ $\overset{\text{INDEP}}{=} \dfrac{\sigma_x}{\sqrt{10}} = \dfrac{0.2}{\sqrt{10}}$

b. Sketch the **exact** normal distribution of $xBAR_{10}$ (applicable since we are sampling from a process in control). Indicate the mean and s.d. of this normal as recognizable numerical entities in your sketch, but do not reduce them.



IN CONTROL ⇒ POPULATION DISTRIBUTION IS NORMAL ⇒ DISTRIBUTION OF $\bar{X}_{10}$ IS EXACTLY NORMAL.

7. Balls will be selected **without replacement** and with equal probability on those then remaining from { R R R R G G Y Y Y }.

4 red 2 green 3 yellow 9 all

a. Determine P(R1 G2 R3) and compare it with P(R1 R2 G3). What principle is illustrated?

ORDER OF DEAL DOES NOT MATTER SO THEY MUST BE EQUAL.

$P(R1\ R2\ G3) = P(R1)\ P(R2|R1)\ P(G3|R1 R2)$

$= 4/9\ \ 3/8\ \ 2/7 = 4\cdot3\cdot2/(9\ 8\ 7)$

$P(R1\ G2\ R3) = 4/9\ \ 2/8\ \ 3/7 = 4\cdot3\cdot2/(9\ 8\ 7)$ SAME!

b. Use the rules of probability to calculate P(R2) by breaking this event down according as R1, G1, or Y1. Do not use any other method. Do not reduce your answer.

$P(R2) = P(R1\,R2) + P(G1\,R2) + P(Y1\,R2)$ TOTAL PROBABILITY

$= P(R1)P(R2|R1) + P(G1)P(R2|G1) + P(Y1)P(R2|Y1)$

$= 4/9\ \ 3/8 + 2/9\ \ 4/8 + 3/9\ \ 4/8$ UNREDUCED.

NOTE: TOTAL IS $\frac{12+8+12}{9\cdot8} = \frac{32}{9\cdot8} = 4/9 = P(R1)$

AS IT SHOULD BE BY "ORDER OF DEAL"