STT 351 **Exam 2**

1. Five pumps are pulled from a process under statistical control. The five pumps average 2.3 gallons per minute with sample s.d. = s_x = 0.4.

a. Write the formula for a 90% confidence interval for μ_x.

$\bar{x} \pm t\ s_x/\sqrt{n}$

t	DF	90% CI
	4	2.132

b. Numerically evaluate (a) for the information given but do not reduce.

$2.3 \pm 2.132\ \dfrac{0.4}{\sqrt{5}}$

c. What is your numerical estimate of the population sd σ_x from the information given? Do not reduce. EST OF σ_x IS $s_x = 0.4$

d. What is your numerical estimate of the sd of the sample mean (i.e. your estimate of $\sigma_{\bar{x}}$) based on the information given? Don't reduce.

EST OF $\sigma_{\bar{x}}$ IS $\dfrac{s_x}{\sqrt{n}} = \dfrac{0.4}{\sqrt{5}}$

e. What $\boxed{\text{performance claim}}$ is made for a 99% confidence interval?

99% OF SAMPLES PRODUCE A .99 CI WHICH COVERS μ_x (EXACT FOR t IF POP^{n} IS NORMAL)

f. For large n, what happens to the width of a CI if n is replaced by 4n?

TERM $\dfrac{1}{\sqrt{4n}} = \dfrac{1}{2}\dfrac{1}{\sqrt{n}}$ SO THE CI IF NARROWER - HALF THE WIDTH

g. What happens to $\dfrac{\bar{x} - \mu_x}{s_x/\sqrt{n}}$ if scores x_i are replaced by $(6\ x_i - 3)$?

REMAINS UNCHANGED.

eg $\dfrac{\overline{6x} - \mu_{6x}}{s_{6x}/\sqrt{n}} = \dfrac{\cancel{6}(\bar{x} - \mu_x)}{\cancel{6}s_x/\sqrt{n}}$ UNCHANGED

ALSO $\dfrac{\overline{x+c} - \mu_{x+c}}{s_{x+c}/\sqrt{n}} = \dfrac{\bar{x} - \mu_x}{s_x/\sqrt{n}}$

2. Sixty with-replacement samples are selected from parts plated with process x. Independently of these, 100 samples are selected from parts plated with process y. The following data apply to a measurement of corrosion resistance

	sample mean	sample sd	sample size
x	24.6	3.7	~~100~~ 60
y	23.8	3.9	~~60~~ 100

a. Numerically determine, but do not reduce, your large-n estimate of the margin of error for $\overline{x} - \overline{y}$.

ESTD MOE $\bar{x}-\bar{y}$ is $1.96\sqrt{s_x^2/n_x + s_y^2/n_y}$

$= 1.96\sqrt{\frac{3.7^2}{60} + 3.9^2/100}$

b. Same as (a) except the samples are without relacement and the population sizes are 600 and 800 for x and y respectively. NEED FPC.'S

$$1.96\sqrt{\frac{3.7^2}{60}\frac{600-60}{600-1} + \frac{3.9^2}{100}\frac{800-100}{800-1}}$$

3. It is desired to estimate the mean of $\boxed{y = \text{burst pressure}}$ for a type of plastic inflatable already in use by the public. A random sample of 50 is selected. The sample is effectively with-replacement since the population size N is so large relative to 50. It is anticipated that another measurement x = time-in-use may have a bearing on this, so both x and y scores are measured for each of the 50. We find sample s.d. are

$s_x = 18.7$ **months** $\qquad s_y = 30$ **psi**

If the sample correlation of x with y is r = 0.8 determine the numerical value of the margin of error of the regression based estimator of μ_y. Do not reduce.

$1.96\sqrt{1-r^2}\ \frac{s_y}{\sqrt{n}} = 1.96\sqrt{1-.8^2}\ \frac{30}{\sqrt{50}}$

4. A random sample of 200 lead seals is selected from production. Of these are 34 found to be defective.

a. Give the formula for a 95% z-based CI for the population fraction of defective seals. $\hat{p} \pm 1.96\ \sqrt{\hat{p}(1-\hat{p})}/\sqrt{n}$

b. Numerically evaluate (a) but do not reduce.

$$\frac{34}{200} \pm 1.96 \sqrt{\frac{34}{200}\ \frac{166}{200}} \Big/ \sqrt{200}$$

5. A 90% bootstrap ci for the population mean, based on a data set named "weights" is given by a call to the function

bootci[mean, weights, 10000, 0.90]

a. Give the call required to generate a 92 % bootstrap ci for the population median but with three times the number of bootstrap replications.

bootci[MEDIAN, WEIGHTS, 30000, 0.92]

b. An admittedly too-small bootstrap run of only 5 bootstrap replicate samples of n produces the following ordered list of values | $\overline{x}^*$ - $\overline{x}$ | :

0.205 0.214 0.216 0.246 0.261

Supposing that the sample mean of the original data is $\overline{x}$ = 44.23 give the 80% bootstrap ci for μ_x based on the above.

$\bar{x} \pm \#^*$ $\quad \#^*$ IS 80TH PERCENTILE OF ABOVE LIST = 0.246

ANS 44.23 ± 0.246

6. Machine processes are scored for x = efficiency. It is desired to obtain a 95% ci for μ_x by the method of $\boxed{\text{proportional stratified sampling}}$ with strata corresponding to three levels of cycling rate.

stratum	**1**	**2**	**3**
stratum size	**1000**	**1000**	**500**

TOTAL N = 2500

a. If a total sample size of 25 is used, how is this allocated among the three strata?

10 FROM 1
10 FROM 2
5 FROM 3

b. The stratum by stratum sample means are

stratum	**1**	**2**	**3**
sample mean	**3.1**	**2.6**	**2.8**

Determine the overall sample mean of all 25. Don't reduce.

$$\frac{10}{25}(3.1) + \frac{10}{25}(2.6) + \frac{5}{25}(2.8)$$

c. The estimated s.d. of $\overline{X}$ (from this stratified sample) is given by $\sqrt{\Sigma_1^3 W_i^2 \frac{s_i^2}{n_i}}$. Give the numerical values of the weights W_i.

$W_1 = \frac{10}{25}$ $W_2 = \frac{10}{25}$ $W_3 = \frac{5}{25}$

7. A maximum likelihood estimator selects the model giving the most probability to what has been seen (the data). By this way of thinking, which model best explains the event R1 R2?

model 1: select two <u>with</u> replacement from [R R G G G]

model 2: select two without replacement from [R R G]

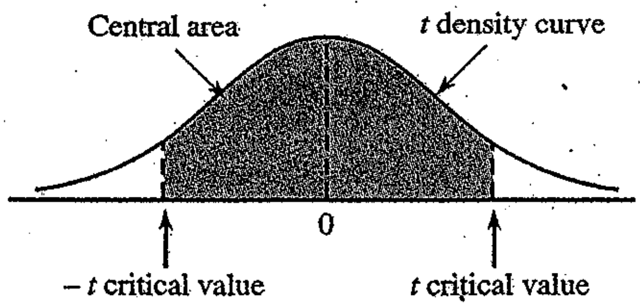
Show your calculations.

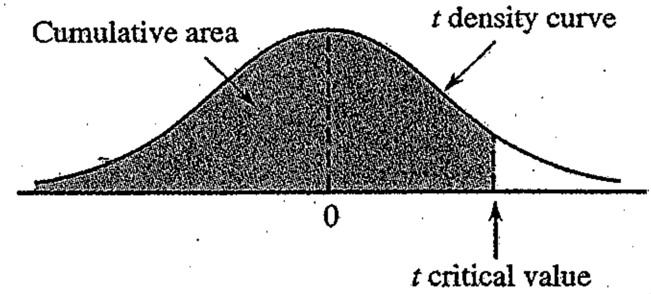
① $P(R_1 R_2) = \frac{2}{5} \frac{2}{5} = \frac{4}{25}$

② $P(R_1 R_2) = \frac{2}{3} \frac{1}{2} = \frac{1}{3}$

$\frac{1}{3} > \frac{4}{25}$ SO MLE CHOICE IS MODEL ②

Table IV *t* critical values for confidence and prediction intervals





Central area = confidence/prediction level for two-sided interval:	80%	90%	95%	98%	99%	99.8%	99.9%
Cumulative area = confidence/prediction level for one-sided interval:	90%	95%	97.5%	99%	99.5%	99.9%	99.95%
Degrees of freedom							
1	3.078	6.314	12.706	31.821	63.657	318.31	636.62
2	1.886	2.920	4.303	6.965	9.925	22.326	31.598
3	1.638	2.353	3.182	4.541	5.841	10.213	12.924
4	1.533	2.132	2.776	3.747	4.604	7.173	8.610
5	1.476	2.015	2.571	3.365	4.032	5.893	6.869
6	1.440	1.943	2.447	3.143	3.707	5.208	5.959
7	1.415	1.895	2.365	2.998	3.499	4.785	5.408
8	1.397	1.860	2.306	2.896	3.355	4.501	5.041
9	1.383	1.833	2.262	2.821	3.250	4.297	4.781
10	1.372	1.812	2.228	2.764	3.169	4.144	4.587
11	1.363	1.796	2.201	2.718	3.106	4.025	4.437
12	1.356	1.782	2.179	2.681	3.055	3.930	4.318
13	1.350	1.771	2.160	2.650	3.012	3.852	4.221
14	1.345	1.761	2.145	2.624	2.977	3.787	4.140
15	1.341	1.753	2.131	2.602	2.947	3.733	4.073
16	1.337	1.746	2.120	2.583	2.921	3.686	4.015
17	1.333	1.740	2.110	2.567	2.898	3.646	3.965
18	1.330	1.734	2.101	2.552	2.878	3.610	3.922
19	1.328	1.729	2.093	2.539	2.861	3.579	3.883
20	1.325	1.725	2.086	2.528	2.845	3.552	3.850
21	1.323	1.721	2.080	2.518	2.831	3.527	3.819
22	1.321	1.717	2.074	2.508	2.819	3.505	3.792