92 004

**1.** The pmf of r.v. ~~X is p(0) = 1/3,~~ p(2) = 2/3. Determine E $\frac{X+2}{X+1}$.

$E\,\frac{X+2}{X+1} = \sum_x \frac{x+2}{x+1}\,p(x) = \frac{2}{1}\left(\frac{1}{3}\right) + \frac{4}{3}\left(\frac{2}{3}\right)$ (YOU NEED NOT REDUCE)

**2.** Random variables X, Y have

EX = 1 sd X = 2 EY = 3 sd Y = 4.

a. Determine E(X + 9Y - 8) $EX + 9EY - 8 = 1 + 9(3) - 8 = 20$ "

b. Determine sd (6 Y - ~~15~~) $|6|\,sd\,Y = 6(4)$ "

c. Supposing that X, Y are independent, determine Variance(10 X + 30 Y + 40).

INDEP $=$ $Var\,10X + Var\,30Y = 10^2 2^2 + 30^2 4^2$ 3

**3.** F stands for "casting is faulty"

\+ stands for "casting appears to be faulty" etc.

P(F) = 0.1 P(+ | F) = 0.6 P(+ | $F^C$) = 0.2

Determine

a. P(+) = $P(F)\,P(+|F) + P(F^c)\,P(+|F^c) = .1\;.6 + .9\;.2$ 3

b. P(F | +) = $P(F+)/P(+) = .1\;.6\,/\,(.1\;.6 + .9\;.2)$ "

**4.** Tom will draw first with equal probability. Sue will draw second with equal probability on those remaining from {5 5 5 5 1 1 1 3 3}

a. P(Sue 5 | ~~Tom~~ Tom 1) (per description, intuitive) DRAW FROM 5555 11 33 $= 4/8$.

b. P(Sue 5) using total probability and mult rules (show your work, do not reduce). $= \frac{4}{9}\frac{3}{8} + \frac{5}{9}\frac{4}{8}$

$P(S5\;T5) + P(S5\;T5^c) = P(T5)P(S5|T5) + P(T5^c)P(S5|T5^c)$

c. P(Tom or Sue get 5) (show your work, do not reduce)

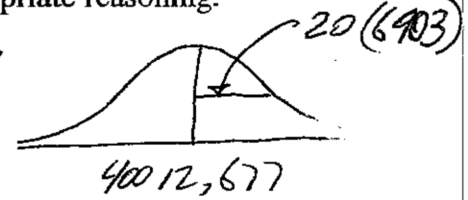
$P(T5 \cup S5) = P(T5) + P(S5) - P(T5\;S5) = \frac{4}{9} + \frac{4}{9} - \frac{4}{9}\frac{3}{8}$

**5.** Business receipts average 12,677 per day with sd 6,903. Receipts on different days seem to be independent. Sketch the approximate distribution of T = total receipts from 400 such days. Be sure to evaluate and display in your sketch E T and sd T. Show appropriate reasoning.

$ET = E(X_1 + \cdots + X_{400}) = 400\,EX = 400(12,677)$

$Var\,T = Var(X_1 + \cdots + X_{400}) = 400\,Var\,X = 400\;6,903^2$

DIST $\approx$ T



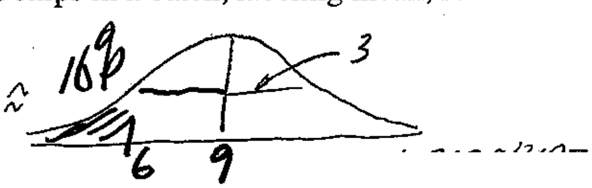

**6.** The Poisson distributed number of bad microchips averages around 9 per batch.

b. How unusual is it to find 6 or fewer bad chips in a batch? (see (a) below)

$\frac{6-9}{3} = -1$ SD $\approx 16\%$ BELOW MEAN $-1.0$ SD.

a. Sketch the approximate dist of number of bad chips in a batch, labeling mean, sd.

SD POISSON $= \sqrt{MEAN} = 3$



Q2 005

**1.** The pmf of r.v. X is p(0) = 1/3, p(-2) = 1/3, p(2) = 1/3. Determine E $X^3$.

$E X^3 = \sum_x x^3 p(x) = 0^3(\frac{1}{3}) + (-2)^3(\frac{1}{3}) + 2^3(\frac{1}{3}) = 0$ (YOU NEED NOT REDUCE TO 0)

**2.** Random variables X, Y have

E X = 2 sd X = 3 E Y = 4 sd Y = 5.

a. Determine E(6 X - Y + 1) $= 6EX - EY + 1 = 6(2) - 4 + 1 = 9$ //

b. Determine sd (11 X + $\cancel{9}$) $= |11|\, SD X = 11(3) = 33$ //

c. Supposing that X, Y are independent, determine Variance(20 X + 15 Y + $\cancel{13}$).

$\overset{INDEP}{=} Var\, 20X + Var\, 15Y = 20^2 Var X + 15^2 Var Y = 20^2 3^2 + 15^2 5^2$

**3.** D stands for "person has the disease"
+ stands for "person tests positive for the disease" etc.

P(D) = 0.2 P(+ | D) = 0.6 P(+ | $D^C$) = 0.1

Determine

a. P(+) $= P(D+) + P(D^c+) = P(D)P(+|D) + P(D^c)P(+|D^c) = .2 \cdot .6 + .8 \cdot .1$

b. P(D | +) $= P(D+)/P(+) = .2 \cdot .6 / (.2 \cdot .6 + .8 \cdot .1)$

**4.** P(A) = 0.5, P(B) = 0.4, P(B | A) = 0.2. Determine but do not reduce:

a. P(AB) $= P(A)P(B|A) = 0.5\ 0.2 = 0.1$

b. P(A∪B) $= P(A) + P(B) - P(AB) = .5 + .4 - .1 = 0.8$ //

c. Are A, B independent? Show your reasoning!

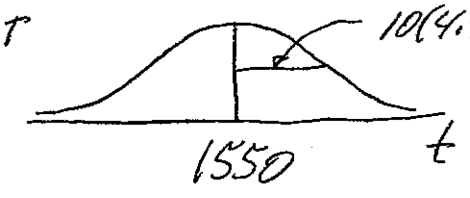
IS $P(B|A) = P(A)$? NO, $.2 \neq .5$. ⇒ A, B ARE NOT INDEPENDENT.

**5.** A random sample of 100 vehicles is selected with replacement and with equal probability from a fleet whose mpg average 15.5 with sd 4.3. Sketch the approximate distribution for T = total mpg of all 100 sample vehicles. Be sure to evaluate and display in your sketch E T and sd T. Show appropriate reasoning.

$ET = E(X_1 + \cdots + X_{100}) = 100\, EX = 1550$

$Var T = Var(X_1 + \cdots + X_{100}) = 100\, Var X = 100\ 4.3^2$

DIST T ≈




**6.** The Poisson distributed number of bad microchips averages around 9 per batch.

b. How unusual is it to find 12 or more bad chips in a batch? (see (a) below)

$\frac{12-9}{3} = +1$ SD 16% ABOVE MEAN + 1.0 SD.

a. Sketch the approximate dist of number of bad chips in a batch, labeling mean, sd.

SD FOR POISSON $= \sqrt{MEAN} = \sqrt{9} = 3$ ≈