STT 315 Practice Problems   Chapter 3.7 and 4

Solve the problem.

1) Suppose that B_1 and B_2 are mutually exclusive and complementary events, such that $P(B_1) = .6$ and $P(B_2) = .4$. Consider another event A such that $P(A \mid B_1) = .2$ and $P(A \mid B_2) = .5$. Find $P(B_1 \mid A)$.

A) .375   B) .240   C) .800   D) .625

2) Suppose the probability of an athlete taking a certain illegal steroid is 10%. A test has been developed to detect this type of steroid and will yield either a positive or negative result. Given that the athlete has taken this steroid, the probability of a positive test result is 0.995. Given that the athlete has not taken this steroid, the probability of a negative test result is 0.992. Given that a positive test result has been observed for an athlete, what is the probability that they have taken this steroid?

A) 0.9928   B) 0.9552   C) 0.0995   D) 0.9325

3) An exit poll during a recent election revealed that 52% of those voting were women and 48% were men. The results also showed that 70% of the women voting favored Democratic candidates while only 40% of the men favored Democratic candidates. These poll results may be summarized as follows:

P(woman) = .52 P(man) = .48
P(favored Democrats | woman) =.70 P(favored Democrats | man) = .40

a. Find P(woman and favored Democrats).
b. Find P(man and favored Democrats).
c. Find P(favored Democrats).
d. Find P(woman | favored Democrats).
e. Find P(man | favored Democrats).

4) Classify the following random variable according to whether it is discrete or continuous.
The number of cups of coffee sold in a cafeteria during lunch

A) continuous   B) discrete

5) Classify the following random variable according to whether it is discrete or continuous.
The height of a player on a basketball team

A) discrete   B) continuous

6) The Fresh Oven Bakery knows that the number of pies it can sell varies from day to day. The owner believes that on 50% of the days she sells 100 pies. On another 25% of the days she sells 150 pies, and she sells 200 pies on the remaining 25% of the days. To make sure she has enough product, the owner bakes 200 pies each day at a cost of $2 each. Assume any pies that go unsold are thrown out at the end of the day. If she sells the pies for $5 each, find the probability distribution for her daily profit.

A)

Profit	P(profit)
$300	.5
$450	.25
$600	.25

B)

Profit	P(profit)
$100	.5
$350	.25
$600	.25

C)

Profit	P(profit)
$500	.5
$750	.25
$1000	.25

D)

Profit	P(profit)
$300	.5
$550	.25
$800	.25

7) A discrete random variable x can assume five possible values: 2, 3, 5, 8, 10. Its probability distribution is shown below. Find the probability that the random variable x is a value greater than 5.

x	2	3	5	8	10
p(x)	0.10	0.20	0.30	0.30	0.10

A) 0.30 B) 0.70 C) 0.60 D) 0.40

8) The random variable *x* represents the number of boys in a family with three children. Assuming that births of boys and girls are equally likely, find the mean and standard deviation for the random variable *x*.

A) mean: 2.25; standard deviation: .87
B) mean: 2.25; standard deviation: .76
C) mean: 1.50; standard deviation: .76
D) mean: 1.50; standard deviation: .87

9) Consider the given discrete probability distribution.

x	1	2	3	4	5
$p(x)$	.1	.2	.2	.3	.2

a. Find $\mu = E(x)$.
b. Find $\sigma = \sqrt{E[(x - \mu)^2]}$.
c. Find the probability that the value of x falls within one standard deviation of the mean. Compare this result to the Empirical Rule.

10) A dice game involves rolling three dice and betting on one of the six numbers that are on the dice. The game costs $7 to play, and you win if the number you bet appears on any of the dice. The distribution for the outcomes of the game (including the profit) is shown below:

Number of dice with your number	Profit	Probability
0	-$7	125/216
1	$7	75/216
2	$9	15/216
3	$21	1/216

Find your expected profit from playing this game.

A) $3.90 B) -$0.92 C) $0.50 D) $7.18

11) A recent article in the paper claims that business ethics are at an all-time low. Reporting on a recent sample, the paper claims that 41% of all employees believe their company president possesses low ethical standards. Suppose 20 of a company's employees are randomly and independently sampled and asked if they believe their company president has low ethical standards and their years of experience at the company. Could the probability distribution for the number of years of experience be modelled by a binomial probability distribution?

A) Yes, the sample size is $n = 20$.
B) No, a binomial distribution requires only two possible outcomes for each experimental unit sampled.
C) Yes, the sample is a random and independent sample.
D) No, the employees would not be considered independent in the present sample.

12) We believe that 95% of the population of all Business Statistics students have taken calculus course. Suppose we randomly and independently selected 21 students from the population. If the true percentage is really 95%, find the probability of observing 20 or more students who have taken a calulus course . Round to six decimal places.

A) 0.283028 B) 0.716972 C) 0.376410 D) 0.340562

Find the probability of the outcome described.

13) Find the probability of at least 2 girls in 10 births. Assume that male and female births are equally likely and that the births are independent events.

A) 0.945 B) 0.011 C) 0.1 D) 0.044 E) 0.989

Solve the problem.

14) A recent article in the paper claims that business ethics are at an all-time low. Reporting on a recent sample, the paper claims that 42% of all employees believe their company president possesses low ethical standards. Suppose 20 of a company's employees are randomly and independently sampled. Assuming the paper's claim is correct, find the probability that more than eight but fewer than 12 of the 20 sampled believe the company's president possesses low ethical standards. Round to six decimal places.

A) 0.662817 B) 0.396113 C) 0.621231 D) 0.260165

15) A literature professor decides to give a 15-question true-false quiz. She wants to choose the passing grade such that the probability of passing a student who guesses on every question is less than .10. What score should be set as the lowest passing grade?

A) 10 B) 9 C) 11 D) 12

16) An automobile manufacturer has determined that 30% of all gas tanks that were installed on its 2002 compact model are defective. If 13 of these cars are independently sampled, what is the probability that more than half need new gas tanks?

17) The probability that an individual is left-handed is 0.13. In a class of 70 students, what is the mean and standard deviation of the number of left-handed students? Round to the nearest hundredth when necessary.

A) mean: 9.1; standard deviation: 2.81 B) mean: 9.1; standard deviation: 3.02
C) mean: 70; standard deviation: 2.81 D) mean: 70; standard deviation: 3.02

18) According to a published study, 1 in every 8 men has been involved in a minor traffic accident. Suppose we have randomly and independently sampled twenty-five men and asked each whether he has been involved in a minor traffic accident. How many of the 25 men do we expect to have never been involved in a minor traffic accident? Round to the nearest whole number.

A) 25 B) 22 C) 3 D) 8

19) The number of road construction projects that take place at any one time in a certain city follows a Poisson distribution with a mean of 5. Find the probability that exactly seven road construction projects are currently taking place in this city.

A) 0.593673 B) 0.104445 C) 0.022469 D) 0.127717

20) The number of traffic accidents that occur on a particular stretch of road during a month follows a Poisson distribution with a mean of 6.5. Find the probability that fewer than three accidents will occur next month on this stretch of road.

A) 0.956964 B) 0.888150 C) 0.111850 D) 0.043036

21) The number of goals scored at each game by a certain hockey team follows a Poisson distribution with a mean of 3 goals per game. Find the probability that the team will score more than three goals during a game.

A) 0.423190 B) 0.576810 C) 0.647232 D) 0.352768

22) Suppose a Poisson probability distribution with $\lambda = 11.9$ provides a good approximation of the distribution of a random variable x. Find σ for x.

A) 141.61 B) 11.9 C) $\sqrt{11.9}$ D) 6

23) Given that x is a hypergeometric random variable, compute $p(x)$ for $N = 6$, $n = 3$, $r = 3$, and $x = 1$.

A) .55 B) .125 C) .375 D) .45

24) Suppose that 4 out of 12 liver transplants done at a hospital will fail within a year. Consider a random sample of 3 of these 12 patients. What is the probability that all 3 patients will result in failed transplants?

A) .018 B) .333 C) .296 D) .037

25) As part of a promotion, both you and your roommate are given free cellular phones from a batch of 13 phones. Unknown to you, four of the phones are faulty and do not work. Find the probability that one of the two phones is faulty.

A) .077 B) .231 C) .462 D) .538

26) Suppose the candidate pool for two appointed positions includes 6 women and 9 men. All candidates were told that the positions were randomly filled. Find the probability that two men are selected to fill the appointed positions.

A) .343 B) .360 C) .160 D) .143

27) You test 4 items from a lot of 15. What is the probability that you will test no defective items if the lot contains 3 defective items?

28) Use the standard normal distribution to find $P(-2.25 < z < 1.25)$.

A) .8821 B) .4878 C) .8944 D) .0122

In a standard Normal model, state what value(s) of z cuts off the described region.

29) the lowest 40%

A) -0.25 B) 0.50 C) -0.57 D) 0.57 E) 0.25

30) the highest 9%

A) 1.45 B) 1.34 C) 1.26 D) -1.34 E) 1.39

Solve the problem.

31) IQ test scores are normally distributed with a mean of 98 and a standard deviation of 18. An individual's IQ score is found to be 114. Find the z-score corresponding to this value.

A) 0.89 B) 1.13 C) -0.89 D) -1.12

Solve the problem. Round to the nearest tenth.

32) Based on the Normal model for car speeds on an old town highway with mean 77 and the standard deviation 9.1, what is the cutoff value for the highest 15% of the speeds?

A) about 11.6 mph
B) about 67.5 mph
C) about 65.5 mph
D) about 63.1 mph
E) about 86.5 mph

33) Based on the Normal model for car speeds on an old town highway with mean 77 and the standard deviation 9.1, what is the cutoff value for the lowest 30% of the speeds?

A) about 53.9 mph
B) about 72.3 mph
C) about 23.1 mph
D) about 60.9 mph
E) about 81.7 mph

34) Based on the Normal model for car speeds on an old town highway with mean 77 and the standard deviation 9.1, what are the cutoff values for the middle 20% of the speeds?

A) about 86.1 mph, about 67.9 mph
B) about 95.2 mph, about 58.8 mph
C) about 61.6 mph, about 92.4 mph
D) about 74.7 mph, about 79.3 mph
E) about 84.7 mph, about 69.3 mph

Solve the problem.

35) A physical fitness association is including the mile run in its secondary-school fitness test. The time for this event for boys in secondary school is known to possess a normal distribution with a mean of 450 seconds and a standard deviation of 50 seconds. Find the probability that a randomly selected boy in secondary school can run the mile in less than 335 seconds.

A) .5107 B) .4893 C) .0107 D) .9893

36) A physical fitness association is including the mile run in its secondary-school fitness test. The time for this event for boys in secondary school is known to possess a normal distribution with a mean of 450 seconds and a standard deviation of 40 seconds. Between what times do we expect approximately 95% of the boys to run the mile?

A) between 0 and 515.824 seconds
B) between 384.2 and 515.824 seconds
C) between 371.6 and 528.4 seconds
D) between 355 and 545 seconds

37) A physical fitness association is including the mile run in its secondary-school fitness test. The time for this event for boys in secondary school is known to possess a normal distribution with a mean of 470 seconds and a standard deviation of 60 seconds. The fitness association wants to recognize the fastest 10% of the boys with certificates of recognition. What time would the boys need to beat in order to earn a certificate of recognition from the fitness association?

A) 371.3 seconds B) 568.7 seconds C) 393.2 seconds D) 546.8 seconds

38) The volume of soda a dispensing machine pours into a 12-ounce can of soda follows a normal distribution with a mean of 12.09 ounces and a standard deviation of 0.06 ounce. The company receives complaints from consumers who actually measure the amount of soda in the cans and claim that the volume is less than the advertised 12 ounces. What proportion of the soda cans contain less than the advertised 12 ounces of soda?

A) .4332 B) .0668 C) .9332 D) .5668

39) The tread life of a particular brand of tire is a random variable best described by a normal distribution with a mean of 60,000 miles and a standard deviation of 2800 miles. What warranty should the company use if they want 96% of the tires to outlast the warranty?

A) 57,200 miles
B) 64,900 miles
C) 62,800 miles
D) 55,100 miles

40) The rate of return for an investment can be described by a normal distribution with mean 42% and standard deviation 3%. What is the probability that the rate of return for the investment exceeds 48%?

41) The board of examiners that administers the real estate broker's examination in a certain state found that the mean score on the test was 553 and the standard deviation was 72. If the board wants to set the passing score so that only the best 10% of all applicants pass, what is the passing score? Assume that the scores are normally distributed.

42) Suppose x is a uniform random variable with $c = 40$ and $d = 50$. Find the probability that a randomly selected observation exceeds 46.

A) 0.6
B) 0.1
C) 0.4
D) 0.9

43) A machine is set to pump cleanser into a process at the rate of 10 gallons per minute. Upon inspection, it is learned that the machine actually pumps cleanser at a rate described by the uniform distribution over the interval 9.5 to 12.5 gallons per minute. What is the probability that at the time the machine is checked it is pumping more than 11.0 gallons per minute?

A) .667
B) .50
C) .25
D) .7692

44) A machine is set to pump cleanser into a process at the rate of 9 gallons per minute. Upon inspection, it is learned that the machine actually pumps cleanser at a rate described by the uniform distribution over the interval 9.0 to 11.0 gallons per minute. Would you expect the machine to pump more than 10.90 gallons per minute?

A) No, since .95 is a high probability.
B) Yes, since .05 is a high probability.
C) No, since .05 is a low probability.
D) Yes, since .95 is a high probability.

45) The diameters of ball bearings produced in a manufacturing process can be described using a uniform distribution over the interval 8.5 to 10.5 millimeters. What is the mean diameter of ball bearings produced in this manufacturing process?

A) 10.5 millimeters
B) 10.0 millimeters
C) 9.5 millimeters
D) 9.0 millimeters

Answer the question True or False.

46) The exponential distribution is sometimes called the waiting-time distribution, because it is used to describe the length of time between occurrences of random events.

A) True
B) False

Solve the problem.

47) Suppose that x has an exponential distribution with $\theta = 1.5$. Find $P(x > 1)$.

A) 0.223130
B) 0.486583
C) 0.776870
D) 0.513417

48) The waiting time (in minutes) between ordering and receiving your meal at a certain restaurant is exponentially distributed with a mean of 10 minutes. The restaurant has a policy that your meal is free if you have to wait more than 25 minutes after ordering. What is the probability of receiving a free meal?

A) 0.082085
B) 0.670320
C) 0.329680
D) 0.917915

49) The time between arrivals at an ATM machine follows an exponential distribution with θ = 10 minutes. Find the probability that more than 25 minutes will pass between arrivals.

A) 0.329680 B) 0.082085 C) 0.670320 D) 0.917915

50) The time (in years) until the first critical-part failure for a certain car is exponentially distributed with a mean of 3.4 years. Find the probability that the time until the first critical-part failure is less than 1 year.

A) 0.033373 B) 0.254811 C) 0.966627 D) 0.745189

1) A
2) D
3) a. P(woman and favored Democrats)
$= P$(woman) × P(favored Democrats | woman) = .52 × .7 = .364
b. P(man and favored Democrats)
$= P$(man) × P(favored Democrats | man) = .48 × .4 = .192
c. P(favored Democrats)
$= P$(woman and favored Democrats) + P(man and favored Democrats)
= .364 + .192 = .556
d. P(woman | favored Democrats)
$= P$(woman and favored Democrats)/ P(favored Democrats)
$= \frac{.364}{.556} \approx .655$
e. P(man | favored Democrats)
$= P$(man and favored Democrats)/ P(favored Democrats)
$= \frac{.192}{.556} \approx .345$
4) B
5) B
6) B
7) D
8) D
9) a. $\mu = E(x) = 1(.1) + 2(.2) + 3(.2) + 4(.3) + 5(.2) = 3.3$

b. $\sigma = \sqrt{2.3^2(.1) + 1.3^2(.2) + 0.3^2(.2) + 0.7^2(.3) + 1.7^2(.2)} \approx 1.27.$

c. $P(\mu - \sigma < x < \mu + \sigma) = P(2.03 < x < 4.57) = .2 + .3 = .5$; The Empirical Rule states that about .68 of the data lie within one standard deviation of the mean for a mound-shaped symmetric distribution. For our distribution, this value is only .5, but it is not a surprise that these numbers aren't closer since our distribution is not symmetric.
10) B
11) B
12) B
13) E
14) B
15) C
16) Let x = the number of the 13 cars with defective gas tanks. Then X is a binomial random variable with $n = 13$ and $p = .30$.

P(more than half) $= P(x > 6.5) = P(x \geq 7) = 1 - P(x \leq 6) = 1 - 0.938 = 0.062$
17) A
18) B
19) B
20) D
21) D
22) C
23) D
24) A
25) C
26) A

27) $P(x = 0) = \dfrac{\binom{3}{0}\binom{12}{4}}{\binom{15}{4}} \approx .363$; $P(x \geq 1) = 1 - P(x = 0) \approx 1 - .363 = .637$
28) A
29) A
30) B
31) A
32) E
33) B
34) D
35) C
36) C
37) C
38) B
39) D
40) Let x be the rate of return. Then x is a normal random variable with $\mu = 42\%$ and $\sigma = 3\%$. To determine the probability that x exceeds 48%, we need to find the z-value for $x = 48\%$.

$$z = \frac{x - \mu}{\sigma} = \frac{48 - 42}{3} = 2$$

$P(x > 48\%) = P(z \geq 2) = .5 - P(0 \leq z \leq 2) = .5 - .4772 = .0228$

41) Let x be a score on this exam. Then x is a normally distributed random variable with $\mu = 553$ and $\sigma = 72$. We want to find the value of x_0, such that $P(x > x_0) = .10$. The z-score for the value $x = x_0$ is

$$z = \frac{x_0 - \mu}{\sigma} = \frac{x_0 - 553}{72}.$$

$$P(x > x_0) = P\left(z > \frac{x_0 - 553}{72}\right) = .10$$

We find $\dfrac{x_0 - 553}{72} \approx 1.28$.

$x_0 - 553 = 1.28(72) \Rightarrow x_0 = 553 + 1.28(72) = 645.16$

42) C
43) B
44) C
45) C
46) A
47) D
48) A
49) B
50) B